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ABSTRACT

The purpose of this study was to test the effect of several cognitive development capacities on first-grade children's ability to learn basic concepts and skills of linear measurement. The cognitive capacities of interest were logical reasoning ability and information processing capacity. The hypotheses predicted that children who had not yet developed the capacities would experience difficulty learning certain measurement concepts or skills. The results confirmed the predictions with respect to logical reasoning ability. Subjects who conserved length and reasoned transitively, performed significantly better than those who did not on the set of assessment tasks which made direct demands on logical reasoning abilities. No significant differences were found between the high and low information processing capacity groups. The results of this study indicate that some cognitive developmental abilities do affect children's mathematics learning, but only on specific concepts. (Author/MK)

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Technical Report No. 506

THE EFFECT OF COGNITIVE DEVELOPMENT ON FIRST GRADE CHILDREN'S ABILITY TO LEARN LINEAR MEASUREMENT CONCEPTS

by

James Hiebert

WISCONSIN RESEARCH AND DEVELOPMENT CENTER
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1025 West Johnson Street Madison, Wisconsin 53706

June 1979

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THE EFFECT OF COGNITIVE DEVELOPMENT ON FIRST GRADE CHILDREN'S ABILITY TO LEARN LINEAR MEASUREMENT CONCEPTS

by

James Hiebert

Report from the Project on
Studies in Mathematics

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Madison, Wisconsin

June 1979

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ABSTRACT

The purpose of this study was to test the effect of several cognitive developmental capacities on first-grade children's ability to learn basic concepts and skills of linear measurement. The cognitive capacities of interest were logical reasoning ability and information processing capacity. The hypotheses predicted that children who had not yet developed these capacities would experience difficulty learning certain measurement concepts or skills.

Potential subjects were pretested to measure their level of cognitive development and to assess their understanding of the measurement concepts on which they would receive instruction. Logical reasoning ability was measured using Piagetian tasks of length conservation and length transitivity, and information processing capacity was assessed with a backward digit span task. The final sample consisted of 32 children who evidenced no understanding of the measurement concepts, with eight children in each cell of a 2 x 2 matrix of high/low logical reasoning ability by high/low information processing capacity.

Similar instruction was provided for all subjects on four basic concepts of linear measurement: 1) using a continuous representation to compare and order two lengths; 2) constructing a discrete representation of a given length; 3) iterating units and representing length numerically; and, 4) accounting for the

inverse relationship between unit size and unit number. Four 10-15 minute lessons, designed to teach the four concepts, were presented to each subject in a one-one interview setting. A unique characteristic of instruction was the introduction of cognitive conflict into the learning situation. After the subject completed a measurement task, the investigator measured in a different way; and the subject was asked to resolve the conflict between the different results.

Several assessment tasks were included as part of each instruction lesson. These were logically analyzed prior to instruction to determine the demands they placed on the logical reasoning abilities and on information processing capacity. This analysis generated predictions about the tasks on which differences between the developmental groups would occur.

Two-way analyses of variance were run on the sets of tasks which presumably made similar demands on logical reasoning ability and those which made similar demands on information processing capacity.

The results confirmed the predictions with respect to logical reasoning ability. Subjects who conserved length and reasoned transitively performed significantly better than those who did not on the set of assessment tasks which made direct demands on these logical reasoning abilities. However, at least some of the subjects who did not possess these abilities performed successfully

on these tasks. Only the final assessment task involving the inverse relationship between unit number and unit size seemed to require conservation and transitive reasoning. No significant between-group differences were found on the complementary set of tasks which only required learning a new measuring skill or technique.

No significant differences were found between the high and low information processing capacity groups. It appears that before significant relationships between processing capacity and mathematics learning can be uncovered, advances must be made in developing analysis procedures which reliably specify the information processing demands of a given task, and devising context-specific measures of processing capacity.

The developmental groups did not differ significantly in their ability to recognize and resolve conflict, but performance in the conflict situations did correlate significantly with performance on the final measurement tasks. No interaction effects between the two developmental factors were found on the measurement task scores or the conflict scores.

The results of this study indicate that some cognitive developmental abilities do affect children's mathematics learning, but only on specific concepts. Future research should systematically document specific relationships between particular developmental abilities and logically related mathematical tasks.

Chapter I

STATEMENT OF THE PROBLEM

Purpose of the Study

A key ingredient in the design of mathematics instruction is the selection of appropriate mathematics content. The learning difficulties experienced by many students studying mathematics demonstrate the importance of prescribing mathematical tasks which are within the learning capabilities of the student. Traditionally, the prescription of content has been based on considerations of mathematical structure, or on general pupil characteristics such as age or grade level. However there is a growing body of research (see Carpenter, in press-a; Case, 1975, 1978a) which suggests that careful attention must be given to specific pupil characteristics if content is to be selected which is appropriate for individual students. These pupil characteristics include certain cognitive developmental capacities which may have a significant effect on children's ability to learn mathematical concepts.

The purpose of this study was to examine the effect of several of these critical mental capacities on first-grade children's ability to learn about linear measurement. The intent of the study was to test directly the hypothesis that cognitive developmental abilities affect children's mathematics learning. Although this hypothesis represents one of the most powerful and fundamental implications of cognitive development for mathematics instruction, it has rarely been tested empirically. Positive correlations between developmental level and mathematics learning have consistently been reported (Carpenter, in press-a), but correlations between two sets of scores provide little insight into the role played by specific developmental abilities in learning particular mathematical concepts or skills. Previous studies have not shown whether specific developmental abilities are necessary to learn certain concepts or skills. It is this kind of information which is needed in order to prescribe mathematical content which is appropriate for individual children. The present study was designed to investigate the effect of two kinds of cognitive developmental capacities on children's ability to learn certain basic concepts and skills of linear measurement.

The purpose of the following discussion is to: 1) describe the nature of the developmental abilities which were investigated and provide a rationale for their inclusion in the study; 2) present a brief overview of the procedures used to test the effect of these abilities on children's mathematics learning; and, 3) create an appropriate context within which to view the contributions and limitations of the study.

Cognitive Developmental Variables

Two basic types of developmental variables were employed in this study. The first deals with children's logical reasoning and is best operationalized within Piaget's theory of cognitive development (Flavell, 1963). The second concerns children's limited capacity to process and integrate information. A relatively novel but potentially useful developmental approach to this problem is described by Pascual-Leone (1970). The following discussion will identify measures of logical reasoning and information processing capacity which are suggested by these theories, and indicate why the measures were used in this study.

Logical reasoning abilities. One of the major contributions of Piaget's work has been the clear demonstration that children's logic is different from that of adults'. Children use qualitatively different reasoning processes than adults to solve certain types of logical problems. The distinction between the logical abilities possessed by children at different stages of development has important ramifications for education. In fact, it is these distinctions which lie at the heart of the potential contributions of Piaget's theory for education in general (e.g., Ault, 1977; Elkind, 1976; Furth, 1970; Hooper, 1968; Schwebel & Raph, 1973; Sigel, 1969; Wadsworth, 1978; and Hooper & DeFrain, Note 1) and for mathematics instruction in particular (e.g., Beilin, 1973, 1976; Copeland, 1974; Lovell, 1966, 1972; Smock, 1973, 1976; Steffe, 1976; and Steffe & Smock, 1975). Most of these attempts to draw implications from

Piagetian theory for educational practice rest on the assumption under investigation here: that children's ability to learn mathematical concepts is influenced by their logical reasoning abilities.

Piaget's theory focuses on the logical thought abilities which are required to solve a variety of tasks. Despite the fact that many Piagetian tasks are mathematically related, it is not clear in what way the mental abilities he identified are required or involved in solving school mathematics problems. For example, at certain levels of development children fail to conserve and fail to use transitive inference. But little is known about how the development of these abilities influence children's learning of related mathematical concepts or operations. From a logical, adult perspective, the absence of these abilities would seem to limit children's ability to learn certain mathematical concepts. However there is ample evidence that children who are preoperational in Piagetian terms can successfully learn and apply a variety of number, measurement, and geometric concepts and skills (see Carpenter, in press-a).

In order to carefully investigate the effect of several Piagetian constructs on children's ability to learn mathematical concepts, this study examined the relationship between length conservation and length transitivity and children's ability to learn several basic concepts of linear measurement. From a logical perspective, conservation and transitivity are prerequisites for measurement. It is difficult to see how lengths of objects can be meaningfully compared or measured if it is believed that simply moving an object, or altering its path, will change its size. According to Piaget, Inhelder, and Szeminska (1960), the absence of conservation precludes measurement, "Underlying all measurement is the notion that an object remains constant in size throughout any change in position " (p. 90). Transitive reasoning is also logically required to measure. All indirect comparisons, as well as unit measurement, require transitive inferences between equalities or order relations (see Steffe & Hirstein, 1976). Clearly the ability to conserve length and reason transitively should affect the kinds of measurement concepts and skills children are able to learn.

Information processing capacity. A well documented principle that has emerged from the study of cognitive development is that young children have a limited capacity to deal simultaneously with several pieces of information (Case, 1978a). Instructional tasks require children to receive, encode, and integrate a certain amount of information. In many cases, children may possess all of the skills presumed to be prerequisites for a particular task and still fail the task. The reason for this failure may be children's restricted capacity to deal with all of the incoming information and their limited ability to integrate the skills which they possess (Case, 1975).

Within a mathematical context, Carpenter (1976, in press-a) and Carpenter and Osborne (1976) suggest that children's difficulty in learning particular concepts may result from excessive information processing demands of the task rather than the absence of logical reasoning abilities. In fact, after reviewing a number of studies on the acquisition of measurement skills, Carpenter (1976) concludes that the variation in children's performance on measurement tasks could potentially be attributed to information processing variables.

Pascual-Leone (1970, 1976) has proposed a theory of cognitive development which focuses on the capacity for processing and integrating information. A central construct of the theory is the limitation associated with the working memory or "M-space" of the cognitive system. It is this component which functions as the information processor, whether the information comes from the external environment or is accessed from long-term memory. The size of this processor or "M-space," where discrete chunks of information are integrated, is considered to be the key ingredient in intellectual development. The basic intellectual limitation is the number of schemes or bits of information which can be handled simultaneously--a capacity that increases regularly with age.

This mental capacity is hypothesized to increase at the rate of one scheme every two years from the preoperational stage (3-4 years) until the late formal operational stage (15-16 years). Consequently, "any general stage of cognitive development could in principle have one numerical characteristic: the number of separate schemes (i.e., separate chunks of information) on which the subject can operate simultaneously using his mental structures" (Pascual-Leone, 1970, p. 302).

Pascual-Leone's theory suggests some possible ways to relate children's developmental abilities to their learning potential in instructional situations. Since much instruction requires students to integrate a number of concepts or skills, information processing capacity or M-space may provide a measure of individual children's ability to benefit from a particular instruction lesson. If the number of elements to be integrated is beyond the capacity of the student, learning will presumably not occur.

The present study examined the effect of this capacity on children's mathematics learning. A backward digit span test was used to measure information processing capacity or M-space. This task has been shown to have a high degree of predictive validity in certain instructional settings (Case, 1974a, 1977).

Description of the Study

The present study was designed to measure the effects of specific cognitive abilities on learning logically related mathematics concepts. Whereas previous work considered only global relationships between these phenomena, this study investigated the effect of specific logical reasoning abilities and an information processing capacity on children's ability to learn linear measurement concepts and skills.

Instruction Content. Mathematics content was selected which is logically related to the developmental reasoning abilities. Since linear measurement is closely tied to length conservation and length transitivity, several fundamental principles of linear measurement were chosen for the instruction sequence. In order to identify and sequence the instruction objectives the basic concepts of linear measurement were analyzed from a psychological perspective. This a priori, theoretically based analysis yielded a framework which identified the important measurement concepts and depicted the relationships between these concepts.

For purposes of this study, linear measurement was thought of as a process of representation. According to Piaget (Piaget et al., 1960), children begin measuring by forming concrete representations of the length attribute of objects. Lengths are first represented using continuous materials. Children are subsequently able to use a series of discrete objects to represent a single length. Eventually they can iterate units, and, with this ability, comes a progression from concrete to symbolic representation. Children can now represent lengths numerically by counting units. Figure 1 outlines this progression by identifying two dimensions of the representation process. One is the move from concrete to symbolic forms of representation; the other is the progression from continuous to discrete to unit iteration as the method of representation. The instruction lessons in this study moved from continuous representation of length to unit iteration and finally to a consideration of the inverse or multiplicative relationship between unit size and unit number.

Although the content of instruction in this study was linear measurement, other mathematical concepts could have been selected to study the effect of cognitive development on children's ability to learn mathematics in an instructional situation. Linear measurement was chosen for several reasons. First, since this study represents an initial inquiry into this question, a topic was selected which would maximize the possibility of relating developmental variables to learning patterns during instruction. Several Piagetian concepts, such as length conservation and length transitivity, are logically tied to linear measurement operations. Consequently an instruction sequence on linear

DEVELOPMENT →

DEVELOPMENT ↓

Representation Level / Method	Concrete	Concrete → Symbolic	Symbolic → Concrete	Symbolic
Continuous	a. Construct a given length b. C & O*2 lengths using an intermediate representation c. Construct a 2nd length equal to the 1st using an inter. representation			
Discrete	a. Construct a given length b. C & O 2 lengths using an intermediate representation c. Construct a 2nd length equal to the 1st using an inter. representation			
Multiple Units		a. Measure a single length and represent it by the number of units b. Measure 2 lengths, C & O them by comparing the number of units	a. Given a measure, construct its length b. C & O 2 measures (given in the same unit) by laying out lengths b'. C & O 2 measures (given in different units) by laying out lengths c. Construct an equal 2nd length by measuring the 1st	
Unit Iteration		a. Measure a single length and represent it by the number of units b. Measure 2 lengths, C & O them by comparing the number of units	a. Given a measure, construct its length b. C & O 2 measures (given in same unit) by laying out lengths b'. C & O 2 measures (given in different units) by laying out lengths c. Construct an equal 2nd length by measuring the 1st	a. C & O measures given same size units but different no. of units b. C & O measures given different size units but same no. of units c. C & O measures given different size units and different no. of units

*Compare and order

Figure 1. Psychological Analysis of Elementary Linear Measurement

measurement seems especially likely as a context in which the hypothesized importance of the developmental factors could be detected.

In order to maximize the possibility of uncovering these relationships it was also necessary to select content which was not confounded with previous school instruction. In most school programs first-grade children are less likely to have received instruction in linear measurement than in beginning arithmetic topics. This means that a sample of first-grade children could be selected who had not yet been exposed to the basic skills and concepts of measurement.

A third reason for selecting measurement as the instructional content was that measurement represents a fundamental concept in elementary school mathematics. Much of early school mathematics can be generated from work with measurement (Romberg, Harvey, Moser,& Montgomery, 1974, 1975, 1976; Van Wagenen, Flora, & Walker, 1976). In spite of this fact, little is known about how children learn to measure. While prenumerical measurement has received considerable attention, the processes by which children begin to assign numbers to measured objects have not been carefully studied (Carpenter, 1976). What is needed at this point is an investigation into how children make use of premeasurement notions to learn measurement concepts (Carpenter & Osborne, 1976).

Sample and Instruction Procedures. The empirical procedures of this study can be partitioned into three major components: sample selection, instruction, and assessment. Each of these will be briefly described. The sample consists of 32 first-grade subjects who were selected according to two criteria: 1) to include children of different developmental levels with respect to length conservation/length transitivity and M-space, and 2) to exclude children who already had some knowledge of the measurement concepts on which instruction was given. The first criterion permitted investigation of the effects of the cognitive abilities on children's learning performance, and the second criterion served in part to equate children in the sample on initial knowledge and to ensure that all children could potentially demonstrate improved performance as a result of instruction. The sample contained an equal number of subjects in each cell of a 2 X 2 matrix of high/low logical reasoning ability by high/low M-space.

After the sample was selected in accordance with these criteria, instruction was provided on several concepts and skills of linear measurement. Children were instructed individually and were actively engaged in measuring by representing, comparing, and constructing various lengths. In addition, they were asked to recognize and resolve the conflict introduced by a discrepant measurement strategy used by the instructor. After the child completed the task (correctly or incorrectly), the instructor measured in a different way and arrived at a different answer in order to produce some conflict which the child was asked to resolve. Cognitive conflict has long been recognized as a powerful motivating force in behavior (Festinger, 1957) and is generally acknowledged to be a potent learning mechanism (Ginsburg & Koslowski, 1976). One reason for its inclusion in these lessons was to maximize the beneficial effects of instruction.

Built into each lesson were several assessment tasks. Thus, children's ability to learn the measurement concepts was evaluated as part of the instruction lessons. This allowed a continuous monitoring of the children's progress and an assessment of the effect of each instructional episode on the relevant measurement concept or skill. The relationship of these assessments to the children's level of cognitive development was described both statistically and anecdotally. A complete description of the procedures is given in Chapter IV, and the particular analyses which were carried out are detailed in Chapter V.

Questions of Interest. The primary purpose of this study was to examine the effect of several cognitive development variables on children's ability to learn measurement concepts. An in-depth analysis of the measurement concepts in terms of the cognitive abilities required to learn them led to several research hypotheses. In general, the hypotheses predicted between-group differences on some measurement tasks and not on others. The logical analyses indicated that some tasks placed heavy demands on certain cognitive abilities, while others did not. Differences between the developmental groups should appear on those tasks which require the particular developmental ability. With respect to the Piagetian constructs of conservation and transitivity, the analysis showed that some of the instruction tasks are logically dependent upon these abilities while others are not; they require only a simple measurement technique. The differences between preoperational and operational children should be greatest on those tasks which require the application of conservation and transitivity principles. Similarly, not all tasks make equivalent demands on information processing capacity. A reasonable hypothesis was that those tasks which require the integration of several measurement skills or concepts demand more capacity for solution than those which focus on a single skill. Therefore the differences between low M-space and high M-space children should be greatest on the tasks requiring skill integration. These hypotheses concerning between-group differences formed the initial questions of interest in this study.

One factor which may help to explain potential between-group differences is children's ability to recognize and resolve the cognitive conflict introduced during instruction. This process may help children reorient their thinking and subsequently improve their understanding of the measurement concept. A question of secondary interest in this study was whether the recognition and resolution of cognitive conflict mediates the relationship between developmental level and ability to benefit from instruction. Are children at a higher developmental level able to recognize and resolve conflict to a greater degree than children at a lower level, and does the recognition and resolution of conflict result in improved performance on the learning tasks? These questions were subjected to empirical test. A precise formulation of the hypotheses and a description of the analysis procedures is given in Chapter IV.

In order to move beyond the determination of statistically significant group differences, the final area of interest focused on the description of processes which children use to complete the measurement tasks. A detailed description of these processes may begin to uncover some important differences in solution strategies which are available to children at different developmental levels. For example, Piagetian operational children are theoretically able to draw upon a qualitatively superior set of mental operations to deal with problem-solving situations.

In this study, children's ability to conserve length and reason transitively should affect the processes they use to solve linear measurement tasks. Descriptive analyses were carried out to 1) characterize the processes used by children at different developmental levels, and 2) characterize the processes used to solve particular tasks which differed in the demands they made on developmental abilities. The intent of these analyses was not only to determine if developmental level makes a difference in what was learned, but also to provide information on how or in what way the developmental capabilities manifested themselves in the learning process.

Rationale and Significance

The significance of this study is best understood if it is viewed within the context of a "linking science" between psychology and education. History has recorded a continuing debate about the effect which psychological theory and research can or should have on education. In an effort to deal systematically with this issue, Glaser (1976) recently revived Dewey's (1900) concern for a linking science between the two disciplines. Glaser argues that the application of descriptive psychological research to prescriptive educational practice cannot rest with the sporadic interests of individual psychologists (e.g., Bruner, 1966; Gagné, 1974, 1977; Skinner, 1968; Thorndike, 1922). If cognitive psychology is going to contribute to instructional programs, a linking science must be established to deal in a systematic and cumulative way with the potential implications of psychological theory and research for education.

Within the area of mathematics education, Carpenter (in press-a, in press-b) supports this view in outlining areas of needed research in mathematics education. Carpenter argues that the unique contribution of mathematics educators vis-a-vis psychologists lies in the construction of a linking science between cognitive psychology and instructional practice in mathematics. Whereas psychological research is only incidentally concerned with learning and teaching school mathematics, research in mathematics education should be aimed directly at significant problems in mathematics instruction. General questions of learning and development should be recast into specific questions about relationships between particular developmental variables and learning school mathematics content. Furthermore, research should focus on the application of current theories of cognition and development to educational practice. Rather than testing and extending psychological theories, mathematics educators should concern themselves with establishing links between existing theories and the learning or teaching of school mathematics.

How does one build a linking science between psychology and education? Speaking in a more general context, Popper (1963) described the process of building sciences as one of testing existing theories, traditions or "myths." Where comprehensive theories have not been established, traditions serve the same function. A science is gradually constructed as traditional notions are scientifically scrutinized and subsequently altered or refined. Since no comprehensive theory exists which outlines the psychological implications for education in general, or for mathematics education in particular, the construction of a linking science will depend, at least in part, on testing traditions.

The significance of this particular study rests with the specific tradition under investigation. A widely accepted and potentially useful belief, which currently holds the status of a tradition in Popper's terms, is that children's level of cognitive development influences their ability to learn mathematics through instruction. Presumably, the rate and course of development are not readily altered by instruction. Qualitatively different mental processes are available to children at different levels of development. Earlier processes are less complete than later ones. Furthermore, earlier levels of development impose certain limits on children's capacity to deal with all of the required information in instructional situations.

Level of cognitive development therefore describes fundamental individual differences between children at a given point in time. The tradition suggests that these individual differences between children can be partially accounted for, and taken advantage of, by providing instructional tasks which are appropriate for each child's level of development.

If the tradition survives scientific test it would have significant consequences for instruction since it would provide a criterion on which to individualize mathematics content. Some form of individualized instruction is the ultimate goal of many instructional models (Klausmeier, Rossmiller, & Saily, 1977). The intent of individualized programs is to provide different children with different types of instructional tasks or a different rate of instruction to maximize its potential benefit for each child. The assumption of these programs is that it is possible to 1) identify characteristics of children which affect their ability to profit from instruction; 2) analyze instructional tasks in terms of these characteristics; and 3) design instruction so that each child receives appropriate tasks in terms of these characteristics.

Traditionally, the student characteristics on which instruction has been individualized have been global measures like IQ or chronological age. True individualization needs to be based on a much more detailed analysis of children's intellectual abilities and the relationship between these abilities and school learning. What is needed at this point for mathematics education is information on the relationship between cognitive development abilities and learning mathematics from instruction. The current study provides this type of information by investigating the effect of several developmental variables on children's ability to learn linear measurement concepts.

Scope of the Study

The previous sections of this chapter have outlined the nature of this study by describing what the study is; this section will provide additional focus for the study by describing what it is not. The purpose of this section is not to detail all of the limitations associated with the methodological procedures--these will be dealt with in the final chapter. The aim is rather to characterize the study by identifying its conceptual parameters.

This study touches on two major fields of research: cognitive development and instruction. In order to clarify the nature of the study it is important to set its boundaries with respect to each of these fields. First, as described in the preceding section, the purpose of this study was to establish links or relationships between cognitive development and mathematics learning. It was a test of potential implications of cognitive development for mathematics instruction, not a validation of developmental constructs. While the study drew heavily from several theories of cognitive development in order to identify relevant cognitive variables, it did not represent a test of the theories themselves. What it did represent was a careful examination of the fundamental contribution which cognitive development holds for improving mathematics instruction.

Second, although the study necessarily employed an instruction procedure it was not a study on instruction. Instructional variables were not systematically manipulated and the outcomes were not explained in terms of these variables. Only one instructional treatment was used, and it differed in significant ways from conventional classroom instruction and other instructional treatments. In other words, the concern of the study was not with the differential effects of different instructional strategies. The study focused on the effects of internal learner characteristics rather than the effects of external instruction procedures.

The general view of instruction adopted in this study is consistent with this emphasis. It is believed that the effects of instruction are mediated in a substantive way by the cognitive processes of the learner. Consequently, an understanding of the instructional process begins with a diagnosis of relevant learner characteristics. As Wittrock (1978) has outlined in his cognitive model of instruction, individual differences among learners are important in the study of learning from instruction, especially the individual differences in cognitive developmental abilities. It was assumed in this study that an examination of the ways in which children with different cognitive characteristics respond to instruction will contribute to an understanding of the instruction/learning process.

It is acknowledged that the study considered only one of the many components of an instructional situation. Carroll (1963) proposed a school learning model with five independent components. The present study investigated an aspect of one of these--appropriateness of the task as a part of the more general notion of quality of instruction. Other important components, such as time allowed to learn the task, were controlled rather than systematically investigated. Changes in these variables may have produced different performance levels. Nevertheless, the argument advanced here is that the nature of the task does represent a key ingredient in instruction; and, given an instruction procedure, the question is whether, and in what way, the developmental abilities of the learner determine its appropriateness. How do cognitive developmental characteristics affect the child's ability to learn certain mathematics content in a particular instruction situation? The relationship or link which was examined in this study can therefore be more specifically identified as a link between cognitive development and instructional content. Carpenter (in press-a) has proposed that this represents one of the most productive areas in which to begin establishing links between the child's developmental abilities and learning mathematics through instruction.

Chapter II

THEORETICAL BACKGROUND

Introduction

Several theories of cognitive development provide the background for the present study. Two of these theories have already been identified and briefly discussed in Chapter I, Piaget's theory and the information processing theory of Pascual-Leone. Two additional theories, Vygotsky's theory of development and the learning theory of Gagné, are also relevant for this study. All four theories directly address the general theoretical notion which underlies this investigation -- the relationship between development and learning. In what way does development constrain, or facilitate, learning? How do the cognitive abilities which emerge with development impinge upon a child's learning potential during instruction? These are the questions which provide the focus for this investigation; and it is the theoretical statements regarding these questions which are of primary interest. This chapter will review aspects of each of the four theories which relate to the questions of learning versus development.

The present study did not represent a test of the theories themselves. However, it did investigate the potential implications of several constructs of these theories for mathematics education. This study drew heavily on these theories in terms of selecting measures of cognitive development, planning instruction procedures, interpreting results, and soliciting general theoretical support for the type of study conducted here. Consequently, a review of the relevant parts of these theories is important for understanding the nature and origins of the theoretical constructs employed in this study. The purpose of this chapter is to provide such a review by characterizing the general notions of learning and development and then briefly describing how each of the four theories views these two concepts. Along with the description will be a rationale for including each theory, i.e., a discussion of how each theory contributes to the current study.

Development and Learning

The terms "development" and "learning" elude meaningful, universal definition. It is difficult to characterize these notions in ways which are acceptable to all four of the theories identified above. The problem is that the theories are based on different assumptions, arise from different world views or paradigms, and consequently define basic terms like development and learning in different ways. These definitions are internally meaningful but are unacceptable to theories based on other assumptions.

A useful distinction between two radically different world views and their categorically-determined theories of development has been proposed by Reese and Overton (1970). One is based on the organismic model and is represented by the theories of Piaget and Vygotsky; the other is based on the mechanistic (or machine) model and is represented by Gagné's theory. Organismic theories believe that it is useful to distinguish between development and learning. While both processes involve changes over time, they are characterized in fundamentally different ways. Development, which is the major concern of these theories, is generally regarded as "a sequential set of changes in the system, yielding relatively permanent but novel increments not only in its structure but in its modes of operation as well" (Nagel, 1957, p. 17). Development is therefore seen to have an effect on the internal structure of the cognitive system resulting in qualitative, as well as quantitative, behavioral changes. Furthermore, genuine developmental events are considered to be those which are universal across individuals and across situations (Wohlwill, 1970a). Changes which are the result of specific experiences or which show up only in certain individuals do not qualify as developmental. Development must therefore be viewed as a broad-based process of change which cannot be accounted for by particular antecedent conditions.

Learning, on the other hand. is thought of in terms of changes occurring under a relatively defined set of conditions, over brief periods of time, and for which antecedent conditions are theoretically specifiable. Learning can result from specific, identifiable conditions and particular learning events can be limited to certain individuals. Learning receives much less emphasis than development in most organismic theories.

Mechanistic theories, on the other hand, are primarily concerned with learning. In fact many such theories define development as the simple accumulation of learning experiences (White, 1970). Particular learning events occur over relatively brief periods of time and can be accounted for by specific antecedent conditions. Development is then defined to be the sum total of all learning events. In this sense development does not constitute a phenomena separate from learning, but is rather subsumed by, or dependent upon, learning.

In summary, both mechanistic and organsimic theories agree that "learning" can be thought of in terms of behavioral changes occurring over relatively short periods of time. The reasons for these changes are theoretically identifiable. While mechanistic theories contend that this type of learning potentially accounts for all changes in human behavior, organismic theories believe it is more useful to postulate another type of change, called "development," which is observable only over longer periods of time and is not reducible to particular environmental causes.

While there are fundamental differences between these theories which may be irreconcilable (Reese & Overton, 1970), many of the differences between the notions of learning and development can be accounted for by the level of analysis of behavior change which is adopted (Wohlwill, 1973). Learning is studied using a microscopic level of analysis to observe changes occurring under a defined set of conditions and over brief periods of time (e.g., minutes, hours, days). In contrast the study of development requires a macroscopic approach where changes are observed (or inferred) in more natural settings and over longer time periods (e.g., months, years).

The current study investigated learning and the effects of development on learning, rather than development itself. Consequently, the study employed a microscopic level of analysis. For this reason it was important to consider the learning principles proposed by those (mechanistic) theories which focus on the learning process. Gagné's (1974, 1977) theory of learning was selected for this study because of its frequent application to instructional settings. It was also important to consider the developmental constructs relevant to learning proposed by those (organismic) theories which focus on the developmental process. For reasons to be outlined below, three organismic theories of cognitive development were selected for review: Piaget's, Pascual-Leone's and Vygotsky's. The remainder of this chapter will briefly present the position of Gagné's learning theory, and each of the three developmental theories, on the question of learning versus development. It will describe how constructs from each theory were incorporated in the study.

Piaget's Theory

For Piaget (1964, 1970, 1974), as for most organismists, there is a clear distinction between learning and development. Cognitive development is a spontaneous process embedded in the context of a developing human system. The development of cognition is inseparable from the growth of biological and psychological faculties. It is a broad-based process, generalizing to a wide variety of specific situations.

Learning, on the other hand, is a limited process. It occurs when provoked by specific external situations (e.g., a didactic point made by an educational experimenter). It is not widely generalizable but is usually restricted to a single problem or concept. This is not to minimize the importance of "learning" since it constitutes an essential part of the educational process (Piaget, 1971b). However, these descriptions portend Piaget's view on the relationship between these two notions.

His position is summarized in the following statement: "I think that development explains learning, and this opinion is contrary to the widely held opinion that development is a sum of discrete learning experiences." (Piaget, 1964, p. 176). The phrase "development explains learning" is more significant and loaded with meaning than it might appear at first glance. It implies that the outcome of a learning experience is accounted for by developmental capabilities. That is, learning potential is defined (or explained) by developmental capacity.

This idea can be clarified by placing it within the context of the Piagetian notions of assimilation and accommodation. For Piaget, development is motivated and controlled by the dynamic tension between these two ubiquitous processes. Simply described, assimilation is the incorporation of external stimuli into existing mental structures. Often, if not always, the external stimuli need to be modified in order to "make sense," or to "fit" the internal mental structues, and thereby become assimilated. Accommodation is the complementary process which involves the modification of mental structures to bring them "in line" with external reality.

A useful, although oversimplified, picture of the interplay of these two processes is the following. Accommodation interjects a qualitatively new mental operation into the cognitive repertoire. Assimilation utilizes this operation in an ever-extending variety of situations to internalize incoming information. This operation becomes inadequate (i.e., it is unable to make sense out of some novel stimulus) and mental restructuring (accommodation) occurs, generating a higher-order mental operation. In cyclic fashion this pattern repeats itself over and over. This narrative is oversimplified because it is difficult to isolate a specific cycle and label the appropriate parts "assimilation" and "accommodation." These processes are active on many fronts simultaneously, and any temporal ordering of them is futile.

However, these concepts are useful in interpreting Piaget's view of learning and development. Learning involves assimilation while development consists of the inter-action of assimilation and accommodation. Since assimilation is dependent on the type of mental operation which is available, it follows that learning is dependent on the developmental stage of the learner. Piaget describes this situation in the following series of statements.

> I shall define assimilation as the integration of any sort of reality into a structure, and it is this assimilation which seems to me to be fundamental in learning, and which

> seems to me to be the fundamental relation from the point of view of pedagogical or didactic applications (Piaget, 1964, p. 185).
>
> No learning occurs when the subjects are too young for there to be a possibility of extending the zone of assimilations. . . . A positive effect is obtained when the aspects introduced by the training constitute an assimilatory instrument, but this is also dependent on the subjects' developmental level, i.e., his competence. . . the notion of competence has to be introduced as a precondition for any learning to take place (Piaget, 1974, pp. xii-xiii).

Any discussion of Piaget's views on learning and development would be incomplete without a description of the distinctions Piaget makes between different types of learning or knowing. These distinctions are important both for understanding Piaget's theoretical position and for applying the theory to an instructional context (Smock, 1976). Furthermore, it is these distinctions between qualitatively different kinds of learning which provide such a marked contrast between Piaget's theory and many well-known learning theories. Piaget makes two types of distinctions, one between operative learning and figurative learning, and another between logical-mathematical knowledge and physical knowledge. These two distinctions are closely related (i.e., operative learning usually involves logical-mathematical knowledge) but they are not synonymous.

The distinction between operative and figurative learning is a distinction between logically-based learning and empirically-based learning (Smock, 1976). It is a distinction between learning about transformations and learning about states (Piaget, 1970). It is a distinction between learning based on the generalizable aspect or "form" of an activity and learning based on the particular aspect or "content" of an activity (Furth, 1969). Operative learning generalizes across content, transfers to related problems, is invariably stable (i.e., is not based on recall), and is resistant to extinction; figurative learning is content specific, is subject to memory loss, and is susceptible to counter-suggestion.

All learning follows the laws of development (Piaget, 1964), but different types of learning "follow development" in different ways. Both figurative and operative learning follow development in the sense that both have developmental prerequisites. However, even here there is a difference. The developmental prerequisites for figurative learning (e.g., perception and memory) are already present at an early age, while those for operative learning (e.g., logical operations) continue to develop throughout childhood and adolescence (Furth, 1969), and are in fact the hallmarks of Piaget's developmental stages. In addition, operative learning follows development in the sense that it proceeds by the same laws or mechanisms which guide development. According to Piaget, the primary mechanism in both cases is equilibration, the dynamic balance between assimilation and accommodation achieved by the recognition and resolution of cognitive conflict. Operative learning, therefore, depends upon developmental abilities for its occurrence and progresses via developmental mechanisms. In many cases it is meaningless to distinguish this type of learning from development itself (Furth, 1969).

A second distinction made by Piaget, which corresponds closely to the first, is between logical-mathematical knowledge and physical knowledge. The first results from acting on objects and discovering properties of the actions; the second results from acting on objects and discovering properties of the objects (Piaget, 1970). The first arises from deduction and is verifiable by logical reasoning; the second arises from induction and is verifiable by empirical test (Beilin, 1976). While logical-mathematical knowledge is generated by internal mental processes, physical knowledge is achieved by direct contact with the external environment via one of the five senses (Steffe, 1976).

Applied to the present study, Piaget's theory on learning and development, and his distinctions between figurative and operative learning and between logical-mathematical and physical knowledge have several significant implications. First, the theory clearly implies that the developmental level of children constrains their ability to benefit from an instructional lesson. Several Piagetian measures of cognitive development which are logically related to linear measurement were included in this study to assess the effect of development on learning measurement concepts. Second, the distinctions between the different types of learning and/or knowledge suggest that the different measurement tasks may require different types of learning and may therefore be differentially affected by the developmental variables. Some of the tasks may require only figurative learning or physical knowledge and may therefore be accessible to many preoperational children. Other tasks may involve logical-mathematical knowledge and require operative learning. These tasks would be mastered only by concrete operational children since it is these children who have attained the logical operations which are theoretically required to achieve operational measurement. The measurement tasks were analyzed to differentiate those based on physical knowledge from those based on logical-mathematical knowledge.

Finally, according to Piaget's theory, operative learning is motivated by equilibration, or the resolution of cognitive conflict (Piaget, 1971a). This mechanism is believed to be responsible for the acquisition of all logical-mathematical concepts. A learning procedure based on this mechanism has been successfully employed by the Genevans in their studies on learning and development (Inhelder, Sinclair, & Bovet, 1974). The effectiveness of this procedure as a learning mechanism in measurement contexts has been recently demonstrated (Carpenter & Hiebert, Note 2). The present study made use of this theoretical construct by designing instruction which introduced conflict into the learning situation. The experimenter posed solutions to the measurement problems which differed from the child's, and the child was asked to explain the difference between the solutions in terms of the measurement principles involved. The alternate solutions provided by the experimenter differed along dimensions which were found in pilot work to have a high level of appeal or salience for the children so that genuine conflict was induced. Of course, according to Piaget, whether or not children experience some form of cognitive conflict when it is introduced into the learning situation is itself dependent upon their level of cognitive development.

Pascual-Leone's Theory

A second major theoretical orientation to be considered in this study emanates from the rapidly expanding field of information processing psychology. Although there are substantial differences between theories, they are all based on the thesis that the input to a psychological processing system, which may be external or internal, provides information that is transformed and acted upon in a variety of ways demanded by the task. An attempt is made to account for performance on cognitive tasks in terms of actions that take place in a temporally ordered flow. Therefore, most theories characterize mental functions in terms of the way information is stored, accessed, and operated upon. Mental structures, on the other hand, are often discussed in terms of an intake register through which information from the environment enters the system, a working or short-term memory in which the actual information processing occurs, and a long-term memory in which all knowledge is stored. A critical structural component of such a system is its short-term memory. It is critical for two reasons: 1) it is extremely limited in capacity, and 2) it is the locus of all processing, whether the information comes from the external environment or is accessed from long-term memory.

The increasing capacity of this working memory, i.e., of the capacity to process information, is a fundamental characteristic of cognitive development (Bruner, 1966; Case, 1978c; Flavell, 1971). Young children are still quite limited in their ability to deal with all of the information demands of complex tasks. This limited capacity may be a critical developmental factor which constrains children's learning in instructional situations (Case, 1975, 1978a, 1978b).

As described in Chapter I, Pascual-Leone (1970, 1976) has proposed a theory which operationalizes the development of this information processing capacity or "M-space." Since this capacity is hypothesized to be the critical factor in cognitive development and serves to identify developmental differences between individuals, and since the ability to process information may be an important variable in instructional situations, the construct of M-space holds significant promise for attempts to relate children's level of development with their ability to profit from instruction. Pascual-Leone's theory is therefore particularly relevant for the present study.

Pascual-Leone's view of the relationship between learning and development is similar to Piaget's. "Not only can intelligence not be reduced to learning, but patterns of learning and the ceiling of learning achievements are a function of the subject's intellectual levels" (Pascual-Leone, Note 3, p. 3). According to Pascual-Leone, learning is a change in behavior resulting from factors which are extrinsic to the psychological system. Within the theory, learning is seen to produce a change in the repertoire of schemes (internally represented behavioral units or patterns) available to the subject. Since M-space is of limited capacity, the number of information chunks which can be coordinated to produce a new scheme is limited, and therefore the complexity of learned schemes is also limited. In this way the processes of learning are constrained by the developing psychological system.

Pascual-Leone's theory provides a potentially useful counterpart to Piaget's theory of development. Piaget emphasizes the structural aspects of development and suggests that learning through instruction depends upon the presence of internal logical operations. Pascual-Leone, on the other hand, is concerned with the functional aspects of development and the temporal mental processing of information; learning through instruction depends on the child's capacity to process all of the essential incoming information. The complementary relationship between these theories has already been empirically demonstrated. Information processing variables have been shown to account for much of the performance variation often found on Piagetian tasks (Baylor & Gascon, 1974; Baylor & LeMoyne, 1975; Case, 1974b; Hamilton & Launay, 1976; Hamilton & Moss, 1974; Parkinson, 1975; Scardamalia, 1977). This study will include a measure of information processing capacity in order to test its usefulness as a measure of children's ability to benefit from instruction.

It must be recognized that applications of Pascual-Leone's theory to instructional settings are still in an exploratory stage. Due to the relatively recent formulation of the theory itself, its implications for education have not been clearly delineated or tested. Several remaining problems prevent a definitive investigation of the role of M-space in instructional situations. The major problem faced by this study is the following. In order to generate hypotheses about children's performance on specific tasks, both the information processing capacity (M-space) of the child and the information processing demands of the task must be known. The first is relatively straightforward since measures of M-space have been developed. But the second is more problematic. Analysis of the task in terms of its information processing demands must be carried out from the child's point of view. "The natural units into which the learner analyzes the task should be considered more important than the a priori units into which a sophisticated instructor might divide them" (Case, 1975, pp. 84-85). This type of analysis is particularly difficult since different children have different schemes available in their cognitive repertoires and hence may approach problems in different ways. "Since M demand is defined from the subject's point of view, the same task may have different M demands for different subjects, depending on the schemes they bring to the task and on how they chunk the information presented to them in the task" (Scardamalia, 1977, p. 29).

To date, most empirical work emanating from Pascual-Leone's theory has employed specially designed novel tasks and a brief pre-training to ensure that 1) all subjects had similar cognitive repertoires with respect to the task, and 2) a task analysis was possible which detailed step-by-step the processes which children could use to solve the tasks (see, e.g., Case, 1972b, 1974a; Parkinson, 1975; Scardmalia, 1977).

At this point it is not clear how such a fine-grained analysis of conventional school mathematics tasks might be carried out. Consequently the approach taken in this investigation was the following. First, children were selected who had a similar knowledge base with respect to measurement concepts, i.e., who had a similar set of schemes available for solving measurement tasks. Second, it was assumed that children would use the individual skills or concepts they learned during instruction to solve the post-instruction task. Since these skills represent newly-learned or non-automated skills they require some M-space for their application. While the instruction focused on a sequence of individual skills or concepts, the post-instruction task required the integration of these skills. Therefore it was assumed that if M-space affects children's ability to learn about linear measurement, this effect would be most pronounced on the post-instruction tasks.

Vygotsky's Theory

> The problems encountered in the psychological analysis of teaching cannot be correctly resolved or even formulated without addressing the relation between learning and development in school-age children. Yet it is the most unclear of all the basic issues on which the application of child development theories to educational processes depends (Vygotsky, 1978, p. 79).

Central to Vygotsky's (1962, 1966, 1978) theory of cognitive development is the relationship between learning and development. Although Vygotsky treated these terms more in line with organismic than mechanistic theories, he rejected what he considered to be the two major and opposing views on the relationship between them. The first, which he ascribed to Piaget, says that learning follows or lags behind development. Since development has a heavy maturational component it is not altered by learning experiences. The opposite view, which Vygotsky attributed to behaviorist psychology, sees learning and development as identical phenomena. Development is only the accumulation of learning experiences.

Vygotsky introduces into this polarity of views an alternative position comprised of several theoretical notions. Two ideas are especially germane: the "zone of proximal development," and the distinction between spontaneous and scientific concepts. The zone of proximal development is defined by Vygotsky (1978) as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). While the actual developmental level describes development which has already been completed, the zone of proximal development characterizes the development which is to come. "What is in the zone of proximal development today will be the actual developmental level tomorrow" (Vygotsky, 1978, p. 87). The critical feature of this construct is that the zone is created by learning experiences. The actual developmental processes lag behind the learning processes and this discrepancy produces the zone of proximal development. Therefore learning is believed to lead, rather than follow, development.

The distinction between spontaneous and scientific concepts is important both for understanding Vygotsky's theory and for applying the theory to instructional contexts. Spontaneous concepts are those which result when the child does the abstracting; scientific concepts are those which result when the abstracting is done by an adult who then transmits them to the child, most often by verbal definition. Spontaneous concepts are drawn from the child's experience and exist independent of any conceptual system; scientific concepts always exist as part of a formal conceptual system. Spontaneous concepts are "nonconscious" in that attention is centered on the object and not on the thought itself; scientific concepts are under intellectual control and may themselves be reflected upon. These distinctions may give the erroneous impression that these types of concepts develop independently without any common ground. The fact is that in the interaction of these concepts the relationship between learning and development can be seen most clearly. Vygotsky suggests that the development of the spontaneous concepts proceeds upward, while the development of scientific concepts proceeds downward. At the point of contact the spontaneous concepts imbue the scientific concepts with experiential meaning and vitality, while the scientific concepts provide an organizational framework or system for the spontaneous concepts. Vygotsky proposes that development involves the growth of spontaneous concepts, and learning the growth of scientific concepts.

For Vygotsky, the implication of these two ideas for instruction is clear. First, instruction should be directed toward the zone of proximal development rather than the actual level of development. "The only good kind of instruction is that which marches ahead of development and leads it; it must be aimed not so much at the ripe as at the ripening functions" (Vygotsky, 1978, p. 104). Second, instruction should be directed toward the scientific aspect of already formed spontaneous concepts. It should provide the spontaneous concepts with a formal conceptual system.

Vygotsky often viewed his work in contrast to Piaget's. He believed that the relationship between school learning (or instruction) and development constituted the basic difference between the theories. "Our disagreement with Piaget centers on one point only, but an important point. He assumes that development and instruction are entirely separate, incommensurate processes" (Vygotsky, 1962, p. 166). It is clear that Vygotsky interprets Piaget as proposing an independent, or more accurately, a unidirectional relationship between learning and development. That is, learning depends on development but development is not affected by learning. Therefore instruction has no effect on development. This interpretation could be understood in two ways: one in terms of the rate of development and the other in terms of the course of development. Although Vygotsky implies that his theory differs from Piaget's on both counts, it appears (based on the Genevans' more recent work) that the theories differ only on the latter, if at all. Due to the date of Vygotsky's writings, his comments about Piaget's theory are necessarily based only on Piaget's early work. Therefore his interpretations do not take into account Piaget's distinction between operative and figurative learning. The first has a close and bi-directional relationship with development (Furth, 1969), while the second has the unidirectional relationship suggested by Vygotsky. If instruction succeeds in provoking or motivating operative learning it will necessarily have at least a short term effect on the rate of development (Inhelder et al., 1974; Sinclair, 1973). Both theories agree that the occurrence of this type of learning depends upon the developmental level already reached by the child. Consider the following statements by Vygotsky:

> With assistance, every child can do more than he can by himself--though only within the limits set by the state of his development. . . . It remains necessary to determine the lowest threshold at which instruction in, say, arithmetic may begin since a certain minimal ripeness of functions is required (Vygotsky, 1962, pp. 103, 104).
>
> A well known and empirically established fact is that learning should be matched in some manner with the child's developmental level (Vygotsky, 1978, p. 85).

These comments imply substantial agreement with Piaget's theory. Furthermore, the zone of proximal development proposed by Vygotsky corresponds in many ways to the "zone of assimilations" described by Piaget (1974) and to his "zone of optimal interest" (Furth, 1969). It appears then that there is substantial agreement between the theories on the relationship between learning and development, at least where rate of development is concerned.

With respect to the course of development the matter is less clear. If Vygotsky believed that instruction could in fact alter the course or direction of development, then his theory would differ in a fundamental way from Piaget's. Piaget's theory depends upon teleological causes to explain development and thus development is believed to move inexorably through an invariant sequence to a predetermined goal. Instruction would not alter its course. Unfortunately Vygotsky's position on this issue is not clear from the translated works so a resolution of this question is not possible.

Due to the unique orientation of Soviet psychology, and the existence of only a few translated works, several questions about Vygotsky's theory need to be answered before its implications for instruction can be tested empirically. Therefore, its relevance for the present study was not in providing developmental measures, prescribing instructional procedures, or suggesting some form of task analysis. Rather, the usefulness of Vygotsky's theory lies in its identification and description of several important ideas. Most relevant for this study is Vygotsky's notion of the zone of proximal development. As a description of children's ability to benefit from instruction given their level of cognitive development, it is the exact theoretical construct which was investigated here. In many ways this study can be viewed as a careful, empirical examination of the zone of proximal development. Vygotsky's theory therefore serves to define and bring into focus some of the central ideas in this study.

Gagné's Theory

Learning theories based on behavioristic (or mechanistic) principles have played a major role in American psychology. The relationship between learning and development is viewed quite differently by these theories than by the developmental theories of Piaget or Pascual-Leone. For Piaget, learning is a function of the cognitive developmental level of the child. Most learning theories, on the other hand, assume the existence of skill hierarchies and suggest that learning is a function of the acquisition of separate skills (Beilin, 1976). Learning is not constrained by the developmental level of the child, only by the absence of prerequisite skills.

The theory of Gagné (1974, 1977) represents a learning theory applied to instructional contexts and so provided the learning theory model for this investigation. In Gagné's theory, development is important only because it represents an increasing accumulation of learning experiences. It is not the developmental level (in the Piagetian sense) which affects the child's ability to master a novel task, but rather the achievement of essential prerequisite skills. "Developmental readiness for learning any new intellectual skill is conceived as the presence of certain relevant subordinate intellectual skills" (Gagné, 1977, p. 145). Consequently, the increased intellectual power exhibited by developing human beings results from the cumulative effects of learning. That is, with the accumulation of learning comes an increase in the likelihood that the subordinate skills for a specific problem will already have been mastered.

For Gagné, these cumulative effects of learning do not include qualitative changes in the learning processes themselves. Although different types of learning are hypothesized, all higher forms are reducible to combinations of lower forms. Achievement of the higher forms is again a matter of mastering the prerequisite lower forms. Consequently, children's learning in instructional situations is not limited by immature or incomplete learning processes.

Gagné's theory has been widely applied to instructional settings. Application of his theory has been based primarily on the concept of "learning hierarchy." Gagné suggests that school learning is directed toward the acquisition of an organized set of intellectual skills. These skills, he says, are "related to each other in the psychological sense that the learning of some are prerequisite to the learning of others" (Gagné, 1977, p. 142). This organization of dependency relationships may be represented as a learning hierarchy.

The learning hierarchy of an instructional objective may be constructed by logically analyzing the terminal objective in terms of what skills are required to reach that objective. In iterative fashion the question "what would one have to know to do that?" generates a map of the individual skills required for mastery and their subordinate/superordinate relationships. Moving from the lower subordinate skills to the higher skills "describes an on-the-average efficient route to the attainment of an organized set of intellectual skills which represents 'understanding' of a topic" (Gagné, 1977, p. 143).

The approach of this study was to control for the learning variables described by Gagné, rather than to investigate their importance by systematically manipulating them. A logical task analysis was carried out for each instructional objective. This specified the prerequisite skills and background knowledge required to master each objective. A homogeneous sample was selected with respect to prior achievement of these skills. Assuming an appropriate hierarchy of skills was identified, differences in performance over the instructional sequence were interpreted to result from factors other than differences in prerequisite skills or knowledge. If children having a similar knowledge base of linear measurement differ in their ability to benefit from instruction on linear measurement, this difference can be attributed to something other than the presence or absence of prerequisite skills. It might rather be attributed to differences in cognitive developmental abilities.

Summary

The specific purpose of the current study was to describe in detail the effect of several cognitive developmental variables on children's ability to learn certain linear measurement concepts. Within a broader context, this study can be viewed as an investigation of the effect of development on learning. The purpose of this chapter was to place the present study in this larger context by reviewing several theoretical positions on the relationship between learning and development. While many theories address this issue, the four theories discussed here were selected for their particular relevance and potential contribution to the present study.

It must be remembered that this investigation did not represent a test of any of these four theories. In fact many of the issues discussed in this chapter relating to mechanistic/organismic distinctions arise from fundamental assumptions and are not subject to empirical test (Reese & Overton, 1970). What the study did represent, however, was a test of several implications of these theories for mathematics education. In particular, it was a test of the "tradition" in the Popperian sense (Popper, 1963) that children's level of cognitive development affects their ability to learn mathematics in an instructional context.

Chapter III

REVIEW OF RESEARCH

Introduction

In testing several potential links between cognitive development and mathematics content for instruction, the current study lies in the interface between psychology and education. Consequently it relates, at least indirectly, to many areas of research in both of these disciplines. The intent of this chapter is to review only those studies which are directly relevant to this investigation. Four categories of research were identified which provide important background information and which serve to sharpen the focus of this study. Empirical studies which fit at least one of these categories are included for review.

The first category of research to be reviewed consists of correlational studies which considered the relationship between cognitive development and mathematics learning. Most of these studies correlated performance on Piagetian tasks with general measures of mathematics learning, usually school achievement. The second category of research includes training studies or teaching experiments which investigated the effect of children's level of cognitive development on their ability to learn certain logical or mathematical concepts through instruction. A major difference between studies in the first category and those in this category is one of status versus intervention. Whereas the correlational studies related developmental level and mathematics knowledge at a given point in time, or after a period of general school instruction, the training studies included an instruction or training procedure and related the effects of instruction with children's initial developmental level. Studies of the second type provide the most relevant data with regard to the question of learning and development, the general issue which underlies this investigation.

The third and fourth categories of research deal specifically with measurement concepts. Studies included in the third category are those which investigated the relationship between children's knowledge of fundamental measurement concepts and their level of development. Like the studies in the first category, these are primarily status studies. Most employed cross-sectional procedures to reveal significant developmental relationships. The final category of research is made up of those studies which were specifically designed to teach concepts of measurement to young children. Although some of the studies to be reviewed here could have been placed in the second category, their direct concern with measurement content justifies treating them separately. These studies indicate the types of measurement concepts young children are able to learn through limited but direct instruction.

To summarize the outline of this chapter, the first two sections will review studies which considered the relationships between developmental abilities and the acquisition of various mathematical and logical concepts, and the second two sections will look at similar studies which were specifically concerned with measurement concepts. The first and third sections consist of status studies while the second and fourth sections consist of intervention studies.

Relationship Between Developmental Level and General Mathematics Learning

Many investigators have taken a rather global approach in studying the relationship between cognitive development and school mathematics learning. A frequent technique is to simply administer a battery of Piagetian tasks and a school achievement test, either concurrently or several months apart. Piagetian task performance is then used as a predictor of present, or future, learning success. A more specific approach, which is employed by some researchers, is to relate performance on certain Piagetian tasks (e.g., number conservation) with achievement in specific areas of the mathematics curriculum (e.g., addition and subtraction problems). Regardless of the approach, almost all of the studies in this area have used Piagetian tasks as measures of cognitive development.

A number of studies have investigated the relationship between Piagetian task performance and arithmetic achievement at a given point in time. Kaminsky (1971) gave second- and third-grade children several Piagetian tasks, an arithmetic achievement test, and an IQ test. A significant correlation was found between developmental level and arithmetic achievement, even with IQ held constant. Riggs and Nelson (1976) used two different forms of a length conservation task with first-grade children. Half of the children received a verbal form of the task, and half received a nonverbal form. Only performance on the verbal task was strongly correlated with arithmetic achievement scores, and this correlation was higher than that between IQ and arithmetic achievement. Rohr (1973) administered several conservation tasks and a mathematics achievement test to third-grade students. Conservation performance was significantly related to achievement, with the highest correlations found for the more advanced conservation tasks. Cathcart (1974) and DeVries (1974) found less correlation than the previous studies between Piagetian task performance and school arithmetic achievement. Cathcart gave second-grade students an arithmetic achievement test and several conservation tasks. Significant correlations were found between these measures in the second-grade but not in the third-grade. The nonsignificant results with the older sample may have been due to ceiling effects on the conservation tasks. The sample in DeVries' (1974) study consisted of bright, average, and mildly retarded children, ages 5-7 years. All children received a comprehensive battery of Piagetian tasks and a standardized achievement test. Correlations did not reach significance in most areas, with a particularly low correlation reported between number conservation and the arithmetic achievement subtest. Interpretations of these results must be made with some caution due to the unique sample which included mentally retarded subjects.

Several studies have used Piagetian task performance as a readiness measure and have tested its usefulness in predicting learning success over an extended period of time. Smith (1974) compared performance on Piagetian tasks with traditional indices (e.g., teacher judgements) for predicting end-of-the-year achievement. Using first-grade students, Smith found that the best predictor of arithmetic achievement was performance on the number component of the Piagetian battery. High correlations between Piagetian task performance and first-grade children's later achievement have also been reported by Nelson (1970). Both a group and an individual test of number and length conservation were significantly correlated with an arithmetic achievement test given several months later.

Several studies have demonstrated that these positive correlations between Piagetian task performance and later arithmetic achievement exist over an extended period of time. For example, Dimitrovsky and Almy (1975), Dodwell (1961), and Kaufman and Kaufman (1972) found that kindergarten children's conservation ability was highly correlated with their arithmetic achievement at the end of first grade; Lunzer, Dolan, and Wilkinson (1976) reported that Piagetian task performance in first grade was a good predictor of arithmetic achievement in second grade; and Freyberg (1966) found that using performance on a Piagetian concept test at ages 5-7 significantly increased the accuracy of using mental age alone in predicting arithmetic achievement two years later. In an extended longitudinal study, Bearison (1975) followed kindergarten children over a four-year period to investigate the relationship between conservation ability in kindergarten with arithmetic achievement in third grade. Some of the subjects had been trained to conserve liquid quantity in kindergarten, some were already natural conservers, and some were nonconservers (see Bearison, 1969). Results showed that early spontaneous conservation was significantly correlated with arithmetic achievement but trained conservation was not. Children who had been trained to conserve in kindergarten did not do better in third grade arithmetic achievement than their later conserving peers. Bearison (1975) concludes that some benefit results from being able to assimilate school instruction at a higher developmental level (as evidenced by the high achievement of the early conservers), but this benefit cannot be induced by early training in specific developmental concepts.

A series of investigators have considered the relationship between particular developmental abilities and children's facility with specific mathematical skills or concepts. Steffe (1970) and LeBlanc (Note 4) observed first-grade children's addition and subtraction skills respectively, and their relationship to number conservation ability. Both found that conservation performance was a significant predictor of arithmetic skill, with low conservation scores associated with especially poor arithmetic scores. LeBlanc also reported that number conservation was a better predictor of subtraction skill than was IQ. Sohns (1974), on the other hand, found only a few significant correlations between number conservation and the subtraction skills of first-, second-, and third-grade children. The fact that these significant correlations occurred for only certain types of problems suggests that slightly different skills or concepts may make different demands on various developmental abilities.

Several studies have considered the relationship between performance on Piaget's class inclusion task and various arithmetic abilities. Howlett (1974) tested first-grade children, who had mastered the relevant number facts, on verbal and written missing addend problems. Class inclusion performance was significantly related to scores on the missing addend test, and evidence from several individual interviews indicated that children at different stages of class inclusion ability used different processes to solve the problems. Two investigators found less of a relationship between class inclusion ability and mathematical concepts. Dodwell (1962) reported no clear relation between class inclusion and fundamental number concepts in 5-8 year old children, and Sohns (1974) found no significant correlations between class inclusion and subtraction abilities of first-, second-, and third-grade children.

Summary. Several conclusions emerge from the studies reviewed in this section. First, level of cognitive development as measured by performance on Piagetian tasks is significantly related to arithmetic achievement. This relationship is maintained even when IQ is held constant. Second, developmental level, particularly conservation ability, is a good predictor of arithmetic achievement up to one, two, and even four years later. Developmental level is generally a better predictor than IQ, and when used with IQ significantly increases the predictability compared with IQ used alone. Furthermore, the benefit of early conservation appears to be a truly developmental one, i.e., it cannot be induced by specific training. Third, it is difficult to tease out the relationships between particular developmental abilities and specific mathematical concepts. Although there is some evidence that number conservation is related to certain arithmetic operations, this relationship may depend upon the particular arithmetic task. Different tasks may make substantially different demands on number conservation ability. The same thing can apparently be said for arithmetic operations and class inclusion ability.

The positive correlations found in most of these studies indicate that a relationship does exist between level of development and ability to benefit from instruction, but they provide little insight into the reason for this relationship. High correlations do not imply causal relations. They do not indicate that developmental abilities are prerequisites for learning arithmetic concepts. The basic question is still whether certain developmental abilities are required to learn specific mathematical concepts. The studies to be reviewed in the next section deal with this question more directly by instructing children on particular concepts and relating learning success to initial developmental level.

Level of Cognitive Development and Learning Through Instruction

The second category of research to be reviewed consists of training studies or teaching experiments which considered directly the effects of children's developmental level on their ability to learn certain mathematical or logical concepts. Three sub-categories of research can be identified here. The first two consist of studies concerned with learning and development in general, while the third focuses specifically on mathematics learning. Studies in the first sub-category were conducted within a Piagetian framework and dealt with training children to acquire logical concepts which are often considered developmental themselves. These studies are usually discussed within the Piagetian training literature and are viewed as attempts to accelerate development. However their unique feature is their direct concern with the effect of initial developmental level on children's ability to make use of instruction or training.

The second sub-category of research falls within an information processing framework and consists of a few recent studies which have investigated the effect of children's information processing capacity on their ability to learn various skills and concepts.

Like those in the first sub-category, these studies were concerned with general questions of learning and development. The difference is that in these studies, development was described in terms of information processing capacity rather than logical reasoning ability. Furthermore, while the studies in the first sub-category examined how well children were able to learn certain Piagetian concepts, the studies in this sub-category considered children's ability to master specially designed information processing tasks.

Studies in the third sub-category dealt with the effect of development on children's ability to learn mathematics concepts. These studies differ from the previous ones in the type of criteria tasks employed. While many of the studies in the first two sub-categories used laboratory type learning tasks, those in this section used tasks drawn from school mathematics curricula.
All of these studies were conducted using Piagetian constructs of development.

The potential fourth sub-category of research and the remaining cell of the matrix would consist of studies which relate information processing capacity to mathematics learning. At present this cell is empty; no studies exist which have carefully examined the effect of information processing capacity on children's ability to learn specific mathematical concepts. An important contribution of the present study is the initial data it provides on this relationship.

Training Piagetian concepts. The intent of this section is not to review the large number of training studies which have attempted to improve children's performance on various Piagetian operations. Reviews of these studies exist elsewhere (see Beilin, 1971; Brainerd, 1973; Strauss, 1972); and as Brainerd (1977) points out, most of these studies provide little valid information on the relationship between learning and development since most do not assess children's initial developmental level independent of their performance on the criteria tasks. There is, however, one general conclusion which emerges from this research which is important for this study. Training, of whatever kind, is not successful with very young children, i.e., a minimum level of development seems to be required for children to benefit from training. As Beilin (1971) notes after a comprehensive review of training research, "No logical or mathematical learning is likely to occur, at least without great difficulty and tenuousness, if the concepts to be learned are far beyond the operational level of the child's available cognitions" (p. 117).

Evidence that developmental level significantly affects learning comes from a number of studies which have instructed children of different developmental levels on certain logical reasoning tasks. Several studies have focused on the ability of children of different ages to learn formal operational concepts (Danner & Day, 1977; Ervin, 1960; Lovell, 1961). The investigators uniformly found that older children learned more than younger children, whether age was measured in mental years (Ervin, 1960) or chronological years (Danner & Day, 1977). This was true even when prerequisite knowledge was controlled (Ervin, 1960). Furthermore, instruction was found to be of little value for the younger subjects, presumably because they had not yet attained a required level of developmental competence and did not have available the appropriate cognitive operations (Danner & Day, 1977; Lovell, 1961). Similar conclusions were reached by Voyat (1973) after finding differential effects of instruction on a concrete operational concept with children of different developmental levels.

Although it may be safe to conclude from these studies that children's level of cognitive development constrains their ability to learn logical concepts, the studies suffer from a basic limitation. Equating development with age means that the concept of development is defined ambiguously and developmental level is measured imprecisely. Different children develop at different rates and consequently have certain cognitive operations available at different ages. The use of age as a measure of development ignores this fact and provides little information on the specific developmental competencies which may affect children's ability to learn particular logical or mathematical concepts.

One set of studies which was directed toward clarifying the nature of these developmental constraints was conducted by the Genevans (Inhelder et al., 1974). These training studies focused on the mechanisms responsible for progression from one developmental stage to the next. Piaget's theory suggests that the primary mechanism by which development proceeds is equilibration, or the resolution of cognitive conflict (Piaget, 1971a). Children apply different strategies to solve similar problems, recognize the conflict of their various solutions, and spontaneously resolve this conflict by constructing higher-order strategies. Cognitive conflict is often created when an already existing mental structure is applied in a less familiar domain. Theoretically, development constrains this process by determining the availability of mental structures which can operate in novel domains.

The studies reported by Inhelder et al. (1974) were designed to investigate this hypothesis by examining how one mental structure or operation might affect the development of another. One study has particular importance for the present investigation. In this study the authors attempted to identify the mechanisms that lead to the development of length conservation. The hypothesis was that, since number conservation is often acquired two to three years earlier than length conservation, development of the latter could be facilitated by exercises in which numerical operations could be used to evaluate length.

Working with a sample of number conservers/length non-conservers, ages five through seven, Inhelder et al. (1974) presented a series of three length activities. All tasks were given to all children individually during three brief sessions. Results on a length conservation posttest showed that most of the children improved their performance from the pretest. These results are interpreted by the Genevans as support for their belief that the acquisition of one logical concept or operation depends upon the existence of other related operations. The tasks in this study were presumed to elicit two types of strategies, one arising from the child's number conservation ability (an already acquired operation) and the other from the child's immature concept of length. These strategies, being incompatible, were presumably resolved by the child during the course of the training resulting in improved length conservation performance. The Genevans suggest that these results show development to be an important precondition for learning.

In a re-examination of this question, Carpenter and Hiebert (Note 2) replicated Inhelder et al.'s (1974) study with several important modifications. Both number conservers and number non-conservers were included in the sample of kindergarten children. In addition, an equal number of children were randomly assigned to one of two treatments. The first treatment included those tasks in which number strategies were frequently applied, and the second treatment consisted of the remaining activities. Consistent with Inhelder et al.'s (1974) procedure, children were asked to complete the tasks but were given no feedback on the correctness of their solutions. The children who improved their length conservation performance on the posttest were about equally divided between number conservers and nonconservers, and between treatment I and treatment II children. Carpenter and Hiebert conclude that number operations do not play the critical role described by the Genevans in acquiring length conservation. Furthermore, children seemed to benefit from the measurement-like activities even though they could not conserve length or number. Based on these results the authors concur with Carpenter's (1976) hypothesis that some children may benefit from instruction in measurement which appears to be beyond their level of cognitive development.

Summary. The evidence provided by these studies suggests that initial developmental level has some effect on children's ability to acquire a particular logical concept via instruction. Children at a higher developmental level usually benefit more from the instruction than those at a lower level. Much of the evidence for this conclusion, however, comes from studies which have used gross measures of development such as chronological age. Consequently it is impossible to isolate the particular developmental abilities which limit children's performance with respect to a specific logical concept. Although Inhelder et al. (1974) defined and measured development more precisely, their failure to include children of different developmental levels restricts the generalizability of their results. Furthermore, the evidence collected by Carpenter and Hiebert (Note 2) suggests that the explanation provided by the Genevans for developmental constraints on learning is inadequate. Finally, although development seems to make a difference, it is not clear what minimum developmental level is required to benefit from instruction on a given concept. What is needed at this point in order to clarify these issues are studies which select a concept for instruction, identify logically related developmental abilities, and instruct children of several different levels with respect to these abilities. The present investigation represented a study of this kind.

Information processing capacity and learning potential. Several studies have recently been conducted to examine the effect of information processing capacity on children's learning potential. Most of these stem from Pascual-Leone's information processing theory. The theory postulates an upper bound on learning established by the child's present level of development. Learning may improve performance on certain tasks but it is theoretically unable to remove the basic constraint imposed by cognitive development, i.e., limited information processing capacity. Case (1974a) summarizes this view:

> According to Pascual-Leone's neo-Piagetian theory of development, a subject's performance on any given cognitive task is a function of three parameters: the mental strategy with which he approaches the task, the demand which the strategy puts on his mental capacity (its M-demand), and the mental capacity which he has available (his M-space).

> Specific learning experiences are assumed to be capable of improving a subject's performance on the task by providing him with a more sophisticated mental strategy for executing it, or in certain instances, by decreasing the M-demand of the strategy which he applies spontaneously. However, specific learning experiences are assumed to be incapable of increasing the size of a subject's M-space (p. 382).

Studies which have empirically tested these hypotheses have used specially designed learning tasks for which it was possible to identify the solution strategies which the subjects had available and to determine the specific strategy with which they approached the task. Given this information, along with the subject's M-space, it is theoretically possible to predict whether a given child will be able to learn the task in question.

Several studies have used age as a measure of M-space to investigate the constraints imposed by this parameter. Parkinson (1975) used a specially designed "Concept Attainment Scoob Task" with 5, 7, 9, and 11 year olds. All subjects were trained to mastery on the prerequisite skills or component schemes of a successful solution strategy. The learning task was then presented as a series of trials with feedback provided on the correctness of each response. The M-demand of the task was systematically altered within each age group. Results confirmed that children could learn to perform successfully only the task forms which had an

M-demand within their range.

Case (1974b) used a version of Piaget's bending rods task with 6 and 8 year old children. All schemes required for solution, except one, were assumed to be available to the subjects. This scheme, a "control of variables" strategy, was taught to half the subjects of each age. Hypotheses were: 1) 6 year olds would not master the task since the M-demand of the task, even with all schemes available, exceeded their M-space; 2) all 8 year olds who had received training would pass the task since they had available all required schemes and a sufficient M-space; 3) some 8 year olds that had not received training would fail the task since they lacked a scheme required for solution. The results confirmed these hypotheses.

Equating age with M-space, although theoretically appropriate, is empirically problematic. Recent work (Lawson, 1976; Carpenter & Hiebert, Note 5) using other measures of M-space, such as backward digit span, has shown that M-space is not perfectly predictable from age. Furthermore, the use of age as a measure of M-space makes it impossible to check whether specific learning experiences are, in fact, "incapable of increasing the size of a subject's M-space" (Case, 1974a, p. 382). It is possible that children's improved performance on the learning tasks resulted from a growth in M-space rather than a change in the repertoire of schemes.

Some evidence that the backward digit span task effectively measures children's learning potential has been reported by Case

(1977). In this study the sample consisted of 5-8 year old children whose M-space was measured using a backward digit span task. The learning task was liquid quantity conservation and the instruction was a cognitive conflict training procedure. The results of the training showed that half of the experimental group and none of the control group improved their conservation performance. The improvement was highly correlated with initial M-space. Case concludes that cognitive conflict procedures effectively induce learning, but this effect is mediated by children's M-space, i.e., by their ability to coordinate cues and perceive the conflict.

A number of studies have focused on the other two "parameters" in Pascual-Leone's theory of learning, i.e., on the decrease in task M-demand which results from learning more efficient solution strategies. Case (1974a) pretested 6, 8, and 10 year old subjects on a specially designed digit placement task. By identifying the solution strategy each subject used and analyzing the M-demand of the task when approached with these strategies, the initial performance of the subjects was found to match that predicted by their M-space. Half of the children in each age group were then instructed on more efficient strategies which would reduce the M-demand of the task by a determined amount. After instruction, the performance of the control children had not changed (i.e., there were no retesting effects) but the performance of the instructed children had improved by the predicted amount.

Case (1972a) investigated the effect of an experimental kindergarten program on children's ability to master special verbal classification and gestural classification tasks. These tasks were analyzed in terms of the schemes which must be coordinated for solution. The instruction program was designed to teach each of the separate schemes to an acceptable level of mastery. The results showed, as predicted, that the performance of the experimental children on the post-instruction tasks was equal to the performance of fourth-grade control children on the verbal tasks, and second-grade control children on the gestural tasks. Case concludes that the instruction allowed the subjects to construct new schemes essential for task solution, reduced the M-demand of the task, and thereby improved performance relative to a control group of kindergarten children. However this learning was subject to a predictable upper bound defined by the initial M-space of the subjects and the M-demands of the tasks after instruction.

A study by Whimbey and Ryan (1969), although conducted outside of Pascual-Leone's theoretical framework, also demonstrated the effect of improved solution strategies. The topic under investigation was the role of short-term memory (a significant component of M-space) in college students' ability to learn syllogistic reasoning problems. Significant correlations were found on pretests of digit span and the reasoning problems, However after training in syllogistic reasoning the correlations disappeared. The authors conclude that training provided subjects with automated skills which brought the short-term memory demand of the task within all subjects' capabilities.

Summary. Two conclusions can be drawn from the information processing studies reviewed in this section. First, instruction designed to improve children's efficiency in information processing can improve children's performance on various kinds of tasks. Second, the amount or complexity of this learning is often constrained by children's M-space or information processing capacity. For specially designed tasks the nature of these constraints can be predicted with impressive accuracy. However it is still not clear how information processing capacity contrains learning in more natural and more complex school instruction contexts. The problem, as noted in Chapter II, is that it is not always possible to analyze, a priori, a complex learning task in the same way a child would and to identify all the relevant schemes which the child brings to the task. This makes it difficult to specify the information processing or M-demand of a given task, which is a necessary step in relating children's M-capacity to their ability to learn the task.

In spite of these problems, Case (1978a, 1978b) has reported several successful attempts to apply principles of the M-space construct in instructional settings. These were basically pilot efforts designed to test the effectiveness of various instruction procedures in reducing M-demands of the learning tasks. However, no attempt was made to control for prerequisite knowledge or to relate learning success to M-space. What is needed at this point is an empirical test of the effect of M-space capacity on children's ability to learn a complex and school-related mathematics task. The present study represented an exploratory test of this kind.

Teaching mathematical concepts. A number of studies have investigated the effect of children's level of development on their ability to learn mathematical concepts or skills. Since the content of learning in these studies is taken from school curricula, the information they provide constitutes the most direct evidence available on the relationship between cognitive development and ability to learn school mathematics content through instruction. As noted previously, all of these studies have been conducted within a Piagetian framework.

Several investigators have used chronological age as a measure of cognitive development and have found that older children learn more than younger children, with a certain minimum age apparently required to benefit from instruction. Carr (1971) tested three groups of kindergarten children on various number concepts. One group had received two years of preschool training, including instruction on number, the second group had received one year, and the third group had received none. Performance was low and results showed no significant between-group differences on number skills. Carr concludes that arithmetic instruction, of the kind offered here, is not effective or meaningful until the child has achieved a cognitive developmental level of "number readiness" and has available the necessary mental operations.

Lovell (1971a, 1971b, 1971c) reports on a series of studies which investigated how well secondary school students were able to learn mathematical concepts such as function, proof, and probability. The details of the subject characteristics, experimental design, and the nature of the instruction are incomplete, but the results of all studies showed a positive correlation between performance and chronological age. Lovell concludes that the relationship between performance and age indicates that certain developmental abilities are required for students to benefit from mathematics instruction.

Limitations associated with the use of chronological age as a measure of development have been discussed previously. Since the subjects in the studies by Carr (1971) and Lovell (1971a, 1971b, 1971c) were not tested for the specific developmental abilities believed to be required on the mathematical tasks, the results only indicate that chronological age, with its many experiential and maturational factors, has an effect on ability to learn mathematics. More precise interpretations are desirable but unwarranted.

Mental age has also been used as a measure of cognitive development. Washburne (1939) directed a series of studies which attempted to identify the mental age required to learn various school arithmetic topics. The assumption was that a certain mental age is most appropriate for learning a given concept. Instruction before this level is reached will be relatively inefficient and nonproductive. In order to identify these "optimum" levels for each topic, a series of studies was conducted over a 10-year period, involving a large number of elementary school teachers and students. The general procedure in all studies was the following. A pretest was administered to measure the students' mental age and their knowledge of the topic to be instructed. Instruction was then provided, over several days or weeks, on a specific arithmetic topic. Achievement was measured by a retention test given six weeks later. Average retention scores were plotted for subjects at each mental age. In most cases, the curves obtained rose steadily with an increase in mental age, reached a critical point, and flattened out. The mental age associated with this critical point was interpreted as the optimal level of development for instruction on the given topic.

Although the intent of these studies falls within the domain of the present investigation, several conceptual and methodological differences reduce the relevance of their findings. A fundamental difference is the use of mental age as a measure of development. The problem is that mental age is a psychometric, rather than theoretic, measure of development (see Elkind, 1971). Although correlations were found between mental age and ability to learn arithmetic concepts or skills, no hypotheses can be advanced which might explicate this relationship. There is nothing specifically about mental age that is logically tied to arithmetic skills on the one hand, and cognitive development on the other, to suggest the reason for the relationship. It is simply an empirical relationship, and any other measure of development might have served as well provided it had the desired psychometric properties (e.g., reliability, individual variation, etc.).

The implication of this is that the results must be interpreted quite narrowly. It is difficult to generalize to other instructional settings which use different methods, teach slightly different topics, or evaluate with different criteria. For example, since the results deal only with mastery of a given topic taught in a given way, nothing can be said about the "optimum" mental age at which to introduce a topic. Without knowing the specific developmental abilities which accounted for these relationships and their logical connections with the arithmetic topics, it is impossible to determine how changes in the situational variables would affect the results. The intent of the current study was not only to establish empirical relationships between developmental measures and learning performance but also to explicate these relationships within a theoretical context.

Several studies have been designed to tease out the effect of particular developmental abilities on learning specific mathematical concepts. Using college students as subjects, Adi (1978) investigated the relationship between developmental level as measured by a paper-and-pencil formal operations task and ability to learn certain related problem-solving processes. Subjects were classified by their performance on the formal operations task into one of three developmental levels from concrete to formal operational. All subjects received the same instruction on solving algebraic equations by two methods: 1) inverting or reversing the sequence of operations; and, 2) compensating for an alteration on one side of the equation by similarly altering the other side. It was hypothesized that the method of inversions developmentally precedes the method of compensations, and that differences between the groups in ability to learn the methods would appear on the compensation problems but not on the inversion problems. Results on an equation solving posttest partially confirmed the hypotheses. The higher developmental group performed significantly better than the lower group on both types of problems, but this difference was substantially greater on the compensation problems due to the relatively poor performance on these problems by the low developmental group. The author concludes that development is a factor in learning mathematical processes, and that its effect depends on the type of process to be learned.

A study by Mpiangu and Gentile (1975) investigated the effect of number conservation on children's ability to learn certain arithmetic skills. Kindergarten children were pretested on an eight-item number conservation test and a four-part arithmetic test. Problems on the arithmetic test involved numbers 0-10, and most of them required rote or point counting skills. (Rote counting consists of recitation of the counting numbers in correct sequence, either forward or backward; point counting involves setting up a one-one correspondence between the counting numbers and a set of markers, and labeling the set with the appropriate cardinal number.) An equal number of children were randomly assigned to the experimental and the control group using a procedure to ensure equal distribution of number conservers and nonconservers. The experimental group received ten 20-minute arithmetic training sessions designed to instruct children on the pretest tasks. The control group received the same amount of instruction on unrelated content. As expected, the results showed a significant main effect for treatment in favor of the experimental group. This effect was obtained using a regression analysis with arithmetic posttest scores regressed on number conservation pretest scores. The regression lines for the experimental and control group were essentially parallel, indicating that, although number nonconservers still performed lower than conservers, they had gained as much from instruction. The authors interpret this as strong evidence that conservation: 1) does not affect children's ability to benefit from mathematics instruction; and, 2) is not a necessary condition for mathematical understanding.

Steffe, Spikes, and Hirstein (Note 6) contested this conclusion after conducting a study which investigated whether two Piagetian constructs, class inclusion and number conservation, were required for young children to learn certain number concepts. Their first-grade sample included an equal number of extensive quantifiers (conservers) and gross quantifiers (nonconservers). These classifications were made on the basis of a pseudo-conservation task in which subjects were required to judge the equality of two static and differently arranged sets of markers. The experimental and control groups were formed by placing half of the children of each quantification ability in each group. Both groups received about 40 hours of arithmetic instruction over a three-month period. The control group received conventional school instruction while the experimental group participated in specially designed activities on classifying, set partitioning, counting, solving addition and subtraction problems, and using hand-held calculators. Most of the activities were directed toward improving the children's counting ability.

After instruction all children were tested on 29 individual measures which were clustered into seven achievement variables. Six of these assessed numerical skills such as working with cardinal and ordinal numbers, solving orally presented addition and subtraction problems with, and without, objects, and counting at the rote, point, and rational levels (rational counting is evidenced by counting-on or counting-back to solve a numerical problem). The results of the study are complex and difficult to summarize. However several of the major findings are the following: a) experimental and control groups did not differ significantly on any of the achievement variables; b) number conservers performed significantly better than number nonconservers on those tasks which required rational counting; c) number conservation was not required to perform tasks solvable by rote counting; d) with special training, number conservation was not required to perform tasks solvable by point counting; e) there was no evidence that class inclusion was a readiness variable for any of the numerical tasks.

From their results the authors conclude that children who differed in their developmental abilities (number conservation or quantification) differed in the benefit they derived from instruction (of either type). The learning experienced by the number conservers was qualitatively different than that of the nonconservers. Number conservers were able to acquire rational counting skills and could apply them to a variety of problems. Nonconservers, on the other hand, demonstrated task specific learning and used rote and point counting procedures. The authors suggest that the conclusions of Mpiangu and Gentile (1975) suffer from overgeneralization. While developmental abilities may not affect the learning of simple skills based on physical knowledge (Piaget, 1964, 1970), they are important for learning skills based on logical-mathematical concepts.

Summary. The evidence reviewed in the preceding studies suggests that children's level of cognitive development does affect their ability to learn mathematical concepts. However these affects may be specific to certain developmental ability/mathematical concept dyads. For example, Mpiangu and Gentile (1975) found that children's performance on a single developmental task does not predict their learning potential in all mathematical situations. Nonconservers could learn simple arithmetic skills. However Steffe et al. (Note 6) demonstrated that a form of number conservation is required to learn more advanced and logically-oriented arithmetic concepts. Furthermore, Adi (1978) showed that people with particular developmental abilities could learn related mathematical skills which were within their range of development, but were unable to learn skills beyond their developmental level. In summary, it appears that the constraining nature of development manifests itself through the limitations imposed by the absence of particular developmental abilities on learning logically related mathematical concepts. The present study investigated this hypothesis.

Relationship Between Developmental Level and Knowledge of Measurement Concepts

The studies which are most relevant for the present investigation are those which have focused on children's learning of measurement concepts. As described briefly in Chapter I, the content of instruction in this study was linear measurement. The objectives of the instruction lessons focused on the initial concepts of measurement dealing with physical and symbolic methods of representation. In particular, the lessons dealt with comparing lengths using continuous, discrete, and numerical representations. Basic concepts of linear measurement which characterize these representation systems, such as the additivity of length, and the inverse or multiplicative relationship between unit size and unit number, formed an integral part of the lessons. The measurement concepts of concern are, therefore, those which bridge the gap between premeasurement concepts, such as conservation and transitivity, and the measurement concepts of a well-developed mathematical system (see Blakers, 1967).

Studies which have investigated the acquisition of these concepts can be partitioned into two categories. The first category consists of status studies which are concerned with the relationships between developmental abilities and knowledge of measurement concepts at a given point in time. These studies will be reviewed in this section. The second category is made up of intervention studies which consider the effects of specific instruction on children's learning of measurement concepts. These will be reviewed in the next section.

A number of studies have investigated the relationship between children's cognitive developmental abilities and their understanding of various measurement concepts. Some of these studies have traced the development of these concepts, either cross-sectionally or longitudinally, and some have considered the relationship between developmental abilities and measurement concepts at a single developmental level. All have been conducted within a Piagetian framework, i.e., the developmental abilities were defined in terms of Piagetian constructs such as conservation and transitivity.

The focus on Piaget's work is understandable since Piaget and associates (Piaget et al., 1960) have proposed the most complete theory of the development of measurement concepts in young children. From their perspective, measurement includes both prenumerical aspects, where objects are compared on the basis of some attribute (e.g., length) without assigning number to the attribute, and numerical aspects introduced by unit iteration. In their studies, children, ages 3-12 years, were asked to carry out both types of measurement in clinical interview situations. Based on the results, Piaget et al. (1960) maintain that children's understanding of measurement develops in stagewise fashion and is closely interrelated with the development of conservation and transitivity. Three major stages are identified with respect to the development of length concepts. In the first stage children do not conserve length and cannot make transitive inferences. They are also incapable of using units to measure. Length is viewed only as a function of endpoints; polygonal or undulating paths between endpoints are ignored. By age 6-7 years most children reach the second major stage. They begin to recognize conservation and transitivity in certain situations and they understand some properties of unit measure. For example, given congruent units, they realize that the length measuring more units is longer. However they fail to account for the size of the unit when noncongruent units are used. The final stage is marked by the achievement of unqualified numerical measurement. By age 8 or 9 years children can conserve length and reason transitively. Soon thereafter they attain the final step in Piagetian measurement--they are able to iterate units and understand the inverse relationship between number of units and unit size.

Piaget et al.'s (1960) position on the relationship between conservation, transitivity, and measurement is clear: "Conservation and transitivity are thus shown to be the first and essential conditions for complete [measurement]" (p. 123). Presumably length conservation and length transitivity are prerequisites for measuring length in a meaningful way.

Further research has shown that, although Piaget et al.'s conclusion may not be incorrect, the relationships between conservation, transitivity, and measurement are more complex than the Genevans' statement would indicate. For example, evidence from two studies suggests that conservation and measurement abilities may interact to each facilitate the development of the other, rather than conservation being required for all measuring activity. The first is a study by Taloumis (1975), who investigated the effect of measurement activities on conservation, and vice versa. Three area conservation tasks and two area measurement tasks were given to children ages 6-9 years. The children were randomly assigned to one of two presentation sequences, conservation-measurement or measurement-conservation. The results showed that in both cases children performed better on the second group of tasks. Taloumis concludes that area conservation does not necessarily precede area measurement, and that measurement activities may facilitate the development of conservation.

In reporting the results of a longitudinal study, Wohlwill (1970b) arrives at a similar conclusion but suggests a more complex relationship between conservation and measurement. The study in question investigated the developmental interrelationships between conservation and measurement concepts. Kindergarten and first-grade children were administered conservation tasks of number, length and liquid quantity. They were also given a set of measurement tasks which required them to compare lengths by direct comparison, by using a physical representation, and by unit iteration. All tasks were administered two additional times over an 18-month period. Results indicated that conservation and measurement were related in rather complex ways. No clear or simple pattern emerged but the data suggested the following interrelationship. The simpler measurement concepts are understood prior to conservation, but the more advanced concepts (e.g., the unit number/unit size relationship) are acquired only after conservation is achieved. Wohlwill (1970b) suggests that in the early stages measurement activities may serve to direct children's attention to the relevant attributes and may facilitate, rather than depend upon, the development of conservation. This conclusion is supported in a conservation training study by Bearison (1969).

A number of studies provide additional insight into the types of measuring behaviors which require conservation abilities and those which do not. Many of these studies have focused on the notion of unit and measuring by unit iteration. A substantial body of evidence collected by Carpenter (1975), Wagman (1975), and Bradbard (Note 7) using liquid quantity, area, and length contexts, respectively, suggests that children can and do make appropriate measurement judgments based on the number of units measured.

Carpenter (1975) administered five types of liquid quantity problems to first- and second-grade students. One was a conservation task and the remaining four were measurement tasks which systematically varied the distinguishability of the comparative unit sizes employed, and the perceptual equality (or inequality) of the initial and final states of the two liquid quantities. Results showed that almost all children recognized that more units implied more quantity, and most children maintained these measurement responses in the face of visually conflicting cues. In fact, number was such a salient cue that there was no significant difference in difficulty between conservation problems, where the distracting cues were visual, and measurement problems, where the distracting cues were numerical.

Consistent with these findings are those reported by Wagman (1975). In order to study the development of area concepts in 8, 10, and 11 year old children, four fundamental axioms of area were initially identified from a mathematical analysis of area. These included the additivity axiom, the area axiom (the unit measure of an area is unique), the congruence axiom (two congruent areas measured with the same unit have equal measures), and the unit postulate (the measure of an area can be derived from its length and width). Tasks were devised to measure an understanding of each axiom. Some tasks were given only to children who could conserve area while others were given to nonconservers as well as conservers. The results from the latter are those which are of interest for the current study. It was found that an understanding of the congruence axiom preceded conservation, while the additivity axiom and conservation developed concurrently. The first finding suggests that children were able to attend to the number of units in a unit measure before they could conserve.

Similar results were also obtained by Bradbard (Note 7) in a length context. Bradbard tested first-, second-, and third-grade children using tasks of prenumerical linear measurement, numerical linear measurement, conservation of length, and transitivity. The results showed a stepwise progression in the development of length concepts similar to that described by Piaget et al. (1960). There were, however, some important differences between Bradbard's results and those of Piaget et al. on the relationship between measurement and conservation and transitivity. The latter two abilities were not found to be prerequisites for engaging in measurement strategies such as unit iteration. Some children who could not conserve or make transitive inferences could successfully measure by iterating units.

While it seems clear that conservation is not required to respond appropriately if congruent units are used (i.e., if the measures are defined by the number of units), there is some evidence to suggest that logical reasoning abilities like conservation may be more heavily implicated in more complex measurement concepts, such as the coordination of unit number and unit size. It is certainly true that these concepts are more difficult for children to understand. In Carpenter's (1975) study with first- and second-grade children, only about half the children realized that the size of the unit affects the result, one-fourth understood the importance of using a constant unit of measure, and only a few were able to infer the inverse relationship between unit size and unit number from the measurement results.

Bailey (1974) found that even third-grade children had difficulty applying the inverse relationship between unit number and unit size to evaluate length. Second- and third-grade children were asked to compare the lengths of two polygonal paths. The two paths consisted of unit segments which varied between paths in comparative size, or number, or both. Complete results are not reported but apparently conservation and transitivity preceded the ability to coordinate unit number and unit size.

Hatano and Ito (1965) suggest a distinction which may help to clarify this complex relationship between conservation and children's measuring behavior. In investigating the development of linear measurement concepts, they identified two basic types of measurement tasks. The first type are those which require the application of a learned measuring technique, such as measuring length with a ruler. The second type are those which require logical inference, such as indirectly comparing the lengths of two objects measured with different-size units. In their study, Hatano and Ito (1965) administered several tasks of each type to first-, second-, and third-grade children. They found that the technique-based tasks were easier than, i.e., were performed prior to, the logical inference tasks. For example, most first-grade children could use a ruler to measure length and could attend to the number of units when iterating. But only about one-third of them could conserve, reason transitively, or coordinate unit number and unit size. These data, which are consistent with the results of the studies just reviewed, suggest that conservation plays a very different role in different types of measuring skills. Conservation is clearly not required to count the number of units when measuring or to make use of numerical results to compare lengths. However it may be more closely tied to the acquisition of other measurement skills, such as coordinating unit number and unit size.

This hypothesis was supported by Carpenter and Lewis (1976) who explored the origins of children's eventual understanding of the inverse relationship between unit number and unit size. First- and second-grade children were given two types of linear and liquid measurement problems. In one type of problem, visually equal quantities were measured with different size units and children were asked to re-compare the quantities. Responding on the basis of number of units alone would lead to conservation-type errors. In the other problem, one of two equal quantities was measured with a visibly larger unit. Children were then asked to predict how many smaller units it would take to measure the other quantity. Results showed that in both the linear and liquid contexts children performed significantly better on the prediction problems than on the comparison problem. The authors conclude that the notion of a compensating or inverse relationship between unit number and unit size develops before it can be applied in measurement situations. Thus, the development of this measurement concept appears to follow the same pattern as that of conservation, where the logical reasoning ability is present before it can be applied in physical situations (see Halford, 1969).

The mechanism which may account for the development of both concepts is described by Carpenter (1975). In this study cited earlier, Carpenter systematically varied the visual and numerical cues by which children might compare two measured quantities. The results showed that visual and numerical cues were equally salient. However children could attend to only one cue at a time. They appeared to focus their attention on a single dominant dimension and ignore other relevant dimensions. Carpenter concludes that the increasing ability of children to both measure and conserve can be explained in terms of an increasing ability to decenter their attention and consider several dimensions simultaneously. The centration of young children presumably accounts for the difficulty they experience in coordinating unit size and unit number and recognizing the inverse relationship between the two.

Only a few studies have investigated the relationship between transitive reasoning and children's ability to measure; and the results are even less conclusive than those involving conservation and measurement. Harris and Singleton (1978) conducted a series of experiments to investigate the role of transitive reasoning in children's measurement. Piaget et al. (1960) reported that young children were unable to make use of a middle term when measuring because they lacked the ability to make transitive inferences. In order to test this conclusion Harris and Singleton modified Piaget et al.'s (1960) task of building towers and administered it to 4 and 6 year old children. They found that even 4 year old children built towers equal in height to a distant one by spontaneously copying the height of a nearer tower which they had been shown was equal. The authors conclude that young children can use a middle term to measure, i.e., they are not logically deficient.

In order to determine why the children in Piaget et al.'s (1960) study had not exhibited this transitive reasoning behavior, Harris and Singleton (1978) ran two additional experiments with children ages 4 and 6 years. In the first, subjects were asked to visually compare the height of several towers. While 6 year olds acknowledged the difficulty in comparing towers separated by some distance, most 4 year olds did not hesitate to make visually-based judgments. In the second experiment, children were asked to build the tower anywhere they pleased. Four year olds were content to build their towers in the original location while 6 year olds moved nearer the standard in order to make more accurate comparisons. The results of these two studies are interpreted to mean that younger children have more confidence in their visual comparisons than older children. The authors conclude that the behavioral deficit of young children in measurement situations results from over-confidence in their visual skills rather than from the absence of transitive reasoning.

While this conclusion was supported in a similar study by Bryant and Kopytynska (1976), it must be viewed with caution. Transitive reasoning ability was never directly assessed in either study and consequently any conclusions concerning its role in the development of measurement concepts are tenuous. The fact that 4 year old children were able to exhibit a primitive form of transitive reasoning behavior in a facilitative context is not surprising (see e.g., Braine, 1959). Furthermore, only a single prenumerical concept of measurement was considered. The real question concerns the role of transitivity in the acquisition of increasingly complex measurement concepts. The results of Bailey (1974) and Hatano and Ito (1965) indicate that transitive reasoning may be important in more advanced measurement concepts.

Summary. The relationship between the developmental abilities of conservation and transitivity and children's knowledge of specific measurement concepts is complex and difficult to establish. Some investigators conclude that conservation and/or transitivity are required to carry out measurement strategies (Bailey, 1974; Piaget et al., 1960), while others suggest that certain measurement strategies precede conservation (Carpenter, 1975; Wagman, 1975; Bradbard, Note 7) and may even facilitate its development (Taloumis, 1975; Wohlwill, 1970b). These mixed results may be explained in part by the different measurement concepts used to test children's abilities. As suggested by Hatano and Ito (1965), some measurement strategies are technique-based while others appear to be more dependent on logical reasoning abilities. Consequently young children have much more difficulty with some concepts than with others. Even with respect to a particular conceptual domain, e.g., unit of measure, children's understanding emerges over an extended period of time. They are able to deal with the number of units of measure at a relatively early stage (Carpenter, 1975), probably before they can conserve (Hatano & Ito, 1965; Wagman, 1975; Bradbard, Note 7). However the ability to deal with the size of the unit and to coordinate unit number and unit size is achieved much later, perhaps after conservation is fully developed (Hatano & Ito, 1965; Piaget et al., 1960; Wohlwill, 1970b). Since different measurement concepts may be differentially related to developmental abilities such as conservation and transitivity, any attempt to investigate the relationship between them will need to consider a wide range of measurement concepts. The current study illustrates this by moving from elementary prenumerical measurement concepts to more advanced numerical measurement concepts in its instructional sequence.

Although the relationship between information processing capacity and children's knowledge of measurement concepts was not investigated directly in any of the studies previously reviewed, some evidence does suggest the following. Children can and do perform poorly in measurement situations even though they possess the required logical abilities (Carpenter & Lewis, 1976). The reason for their inability to measure may be explained in part by their inability to decenter and consider several dimensions simultaneously (Carpenter, 1975). The current study investigated this hypothesis by including M-space as a measure of children's information processing or decentering capacity.

The studies reviewed in this section are status studies; most of them employed cross-sectional rather than longitudinal methods of investigation. Therefore, conclusions, based on these results, about the role played by conservation and transitivity in the acquisition of measurement concepts are highly inferential. For example, the fact that children spontaneously develop conservation and/or transitive reasoning before they master the inverse relationship between unit number and unit size does not necessarily imply that these logical reasoning abilities are required for, or even contribute to, the acquisition of this measurement concept during instruction. The evidence provided by status studies is primarily suggestive. More direct evidence about the relationship between developmental abilities and learning measurement concepts comes from intervention or instructional studies.

Acquisition of Measurement Concepts Through Instruction

The purpose of this section of the review is to consider studies which were designed to teach young children the fundamental concepts of measurement. Those to be reviewed have included instruction on the kinds of measurement concepts which are of interest in the current investigation. Therefore the review will include neither studies designed to teach the premeasurement concepts of conservation and transitivity, nor those which instructed older children in more advanced measurement techniques, e.g., using standard units of measure (for a more complete review of measurement studies see Carpenter, 1976). Methodologically, the studies to be reviewed in this section are of two types. One type considered the extent to which children of a particular age or developmental level could learn various measurement concepts. The second type of study investigated the effect of children's developmental status on their ability to learn measurement concepts and consequently included children of different developmental levels.

Several studies of the first type have examined the feasability of teaching initial mathematics concepts through a measurement approach. Van Wagenen et al. (1976) provided an experimental group of first-grade children with an alternative mathematics program for the entire year which emphasized concepts of linear measure. Activities included representing length physically and symbolically, and comparing lengths using various representations. At the end of the year the experimental group performed significantly better than a control group, which had received conventional instruction, on a measurement test and equally well on an arithmetic achievement test.

Another mathematics program, Developing Mathematical Processes (Romberg et al., 1974, 1975, 1976), uses measurement ideas to introduce basic mathematics concepts. Before developing the program, a series of pilot tryouts were conducted to ascertain what measurement concepts young children were able to learn. It was found that after several weeks of instruction, kindergarten children could represent length using continuous physical representations and could compare and order lengths using these representations (Romberg & Gornowicz, Note 8). First-grade children could, after several lessons, measure length using both a collection of congruent units and unit iteration (Romberg & Planert, Note 9); and second-grade children were able to learn the multiplicative relationship between unit size and unit number (Romberg & Planert, Note 10).

A study by Minskaya (1975) focused specifically on introducing the concept of number through measurement activities. First-grade children were presented with specially designed mathematics lessons which dealt with comparing quantities in various ways. Number was introduced during the second half of the year by using unit iteration and number line activities. The emphasis of most lessons was on unit size, and the relationship between total quantity, unit number, and unit size. The results which are reported are impressive. On the final retention and transfer test, which included many problems on the inverse relationship between unit number and unit size, 83% of the responses were correct. This is noteworthy in light of the relative difficulty of this concept for young children, a fact which has been documented in several studies reviewed earlier.

Apparently young children can learn measurement concepts, and can be taught other mathematical concepts through a measurement approach. However this does not mean that children experience no difficulty in learning about measurement. A teaching experiment by Gal'perin and Georgiev (1969) attempted to alleviate some of these problems. Based on earlier work, they hypothesized that children's misconceptions of measurement result from a lack of understanding of the unit of measure. Presumably children do not appreciate the size of units and rely only on visual cues when measuring. In order to correct these misconceptions they devised a series of 68 instruction lessons for 6 and 7 year olds which focused on the measurement process and systematically differentiated between units of measure and discrete entities. The results show impressive gains in children's understanding of measurement.

Although it is clear that the children did learn some basic concepts of measurement the reasons for their initial misconceptions are less clear. Carpenter (Note 11) readministered some of Gal'perin and Georgiev's tasks along with some additional tasks to investigate the reason for children's errors. Carpenter found the same errors as those reported by Gal'perin and Georgiev but based on the additional results suggests an alternate interpretation. Carpenter maintains that the errors are not the result of inattention to the size of units but rather children's limited capacity to make more than one-dimensional comparisons. They are capable of attending to the size of units, or the number of units, but are unable to coordinate these two dimensions.

The previous studies indicate that young children can learn measurement concepts through careful instruction, but some misconceptions may occur along the way. From a logical perspective, these misconceptions may result in part from the absence of fully developed logical reasoning abilities. However, only a few studies have been conducted to test this hypothesis, i.e., to examine the effect of developmental abilities on children's measurement learning.

One such study was conducted by Beilin and Franklin (1962). They investigated the age-related developmental limits imposed upon the acquisition of measurement concepts. First-grade and third-grade students were pretested on their ability to compare and order various objects by length and area using unit iteration.

The children were then instructed on how to solve these tasks using appropriate iteration strategies. The posttest consisted of a series of transfer tasks which used different-size shapes but required the same measurement processes. The results showed that the third-grade children knew more about measurement initially, and were able to transfer their knowledge to the novel tasks on the posttest. The first-grade children displayed some knowledge of linear measurement after instruction, but were unable to learn about area measure. "This lends support to the view that the child's level of development places a limit on what he may acquire by virtue of experience or training at a particular time" (Beilin & Franklin, 1962, p. 618). Unfortunately, the conclusion of the authors is attenuated by the lack of control for prior knowledge, and the use of age as a gross measure of development. It is difficult to assess the effects of specifically "developmental" abilities in this situation, and it is impossible to isolate the particular abilities which may have come into play.

One of these methodological problems, a difference between age groups in prior knowledge, was incidentally alleviated in a study reported by Montgomery (1973). It was not Montgomery's intention to investigate the relationship between developmental status and ability to benefit from instruction in linear measurement, but the study provides significant information in this regard. The purpose of Montgomery's study was to examine the interaction of second- and third-grade children's ability to learn length concepts with two instructional treatments on area concepts. An initial teach-test procedure partitioned subjects on their ability to learn to compare two lengths measured with different units. They were randomly assigned to one of two instructional sequences which differed in their treatment of the unit of area measure. One sequence always used congruent units to compare regions, while the other used noncongruent units and therefore emphasized unit size as well as unit number in all comparisons. It was hypothesized that the children who experienced more difficulty using different units during the teach-test procedure would not be able to take advantage of the latter, more sophisticated instruction. However, the results showed none of the hypothesized interactions. Main effects were found favoring the students who had scored higher on the teach-test assessment, and favoring the instruction sequence which used noncongruent units.

Two results of Montgomery's study are particularly relevant for the present investigation. First, the lack of interaction and the main effect favoring the more advanced instruction sequence indicates that the less capable children benefitted more from this form of instruction, even though it was logically beyond their abilities. This may have been because it directed their attention to all of the relevant attributes in measuring, a general hypothesis suggested by Gelman (1969) in number contexts, and Carpenter and Hiebert (Note 2) in measurement contexts. The main effect for ability level is less relevant for this study since the form of ability which was measured, while of general interest (see Carpenter, in press b), is not a developmental ability of the type assessed here.

The second result of interest pertains to the relationship between developmental status (as measured by age) and ability to learn linear measurement concepts. As mentioned earlier, this information was not an intended outcome of Montgomery's (1973) study and the data were not analyzed with this in mind. However a re-analysis of the original data contained in Montgomery (Note 12) shows the following. Second- and third-grade students did not differ in their knowlege of unit of length concepts before the teach-test procedure. After two periods of instruction, however, the third-grade students performed significantly better than the second-grade students on a unit of length posttest. Evidently the older children were able to benefit more from instruction on length concepts than the younger children. Of course it is still not known what particular developmental abilities were involved.

Summary. It is clear that young children can be taught some of the basic concepts of measurement. But what about the effect of cognitive development on children's ability to learn measurement concepts? Some evidence suggests that children can benefit from measurement instruction which would appear to be beyond their capabilities (Montgomery, 1973; Carpenter & Hiebert, Note 2; see also a conservation training study by Bearison, 1969). However this is not to say that children of all developmental levels are equally able to benefit from instruction (Beilin &

Franklin, 1962; Montgomery, Note 12). Developmental constraints seem to be real, but these may be in the form of information processing limitations, as suggested by Carpenter's (Note 11) results, rather than logical reasoning deficiencies. This is the conclusion arrived at by Carpenter (1976) after an extensive review of measurement research.

Unfortunately, the constraints of cognitive development have usually been inferred from differences in performance of children of different ages. No evidence exists which might link specific developmental abilities to children's learning during instruction on measurement. The current study was intended to fill this gap by investigating the effect of several logical reasoning abilities (conservation and transitivity) and an information processing capacity (M-space) on children's ability to learn linear measurement concepts during instruction.

Conclusions

Previous research has uncovered a significant relationship between cognitive development and mathematics learning. It appears that children's level of cognitive development affects the kinds of mathematical concepts or skills which they are able to learn. An absence of certain developmental abilities limits children's learning potential.

With respect to the Piagetian logical reasoning abilities, the relationship between development and learning seems to be specific rather than general. Certain abilities, such as conservation and transitivity, are required to learn some mathematical concepts but not others. More specifically, length conservation and length transitivity play an important role in learning some measurement concepts, but are not required for learning others. The present study fits well within this research background. It was designed to systematically investigate the effect of length conservation and length transitivity on learning a sequence of increasingly complex measurement concepts and skills.

Much less is known about the relationship between the development of information processing capacity and mathematics learning. Data from laboratory-type settings suggest that this developmental capacity has a direct effect on children's learning potential. In addition, there is some indication that the cognitive capacity which limits children's mathematics learning is the ability to process and coordinate several pieces of information simultaneously. However there is almost no information on the relationship between this ability and learning school mathematics concepts. The present study explored this relationship by applying Pascual-Leone's (1970) notion of M-space to the learning of linear measurement concepts.

Chapter IV

HYPOTHESES AND PROCEDURES

Introduction

The purpose of this study was to examine the effect of basic developmental capacities on children's ability to learn mathematics through instruction. The research strategy employed was outlined briefly in Chapter I and will be expanded upon here. The major steps in this strategy can be summarized as follows. Cognitive developmental abilities were identified for their potential influence on mathematics learning. Mathematics content which is logically related to the developmental abilities was selected for instruction. Since the intent of the study was to isolate and describe the role of these abilities in learning mathematics, the mathematics tasks were logically analyzed to determine their prerequisite skills and to specify the demands they placed on each developmental ability. The results of this analysis were used in two ways. First, the learning hierarchies established through the specification of prerequisite skills were used to design the instruction lessons. Second, the identification of the requirements placed on the developmental abilities by each mathematics task generated hypotheses about the points during instruction where developmental differences would affect learning. The data-gathering process was then implemented by selecting subjects who differed in their developmental status but not in their initial knowledge of the primary measurement concepts. All subjects were provided with similar instruction and their responses to instruction were carefully recorded. Children's performance over the course of instruction was then related to their initial level of development.

The first step in this sequence, the selection of developmental variables, has been disucssed in Chapters I and II. The remaining steps have been outlined previously but will be described in detail in this chapter.

Selection and Analysis of Instructional Content

Linear measurement was selected for instruction because of its logical relationship with the developmental variables and its importance in school mathematics programs. As described in Chapter I, an analysis was carried out to identify and sequence the instruction objectives which embodied the basic concepts of elementary measurement (see Figure 1). Four principles of linear measurement were selected from this analysis, each providing the focus for one instruction lesson. Given in the sequence in which they were presented, the four objectives are: 1) using an intermediary, continuous representation and attending to endpoints when measuring; 2) using a discrete representation (i.e., subdivision of length) and attending to the additivity of length and the linearity of the interval between the endpoints when measuring; 3) using a collection of units (i.e., several same-size discrete objects) and unit iteration (i.e., a single unit and change of position) to assign numbers to lengths; and 4) accounting for the inverse or multiplicative relationship between unit size and number of units when measuring.

Tasks were constructed to assess children's understanding of these principles and these tasks were then carefully analyzed along several different lines. The primary analysis procedure was a rational task analysis as prescribed by Gagné (1977). Each of the four major tasks was broken down into a hierarchy of logically related prerequisite skills as shown in Appendix A. These learning hierarchies identified the skills and concepts required to complete each task. An important characteristic of this type of analysis is the specification of instructable components. Each prerequisite skill represents an intermediate instructional objective. From the standpoint of many learning theories, including Gagné's (1974, 1977), a maximally effective instruction procedure must attend to these prerequisite skills and their hierarchical status.

In this study the highest level prerequisites formed the basis for the instruction lessons and the lower level prerequisites were used to screen subjects on the linear measurement pretest (see

Appendix A). Each lesson consisted of several instruction problems which focused on a single prerequisite and a post-instruction task which embodied the primary concept or terminal objective of that lesson. The analyses in Appendix A identify both the prerequisites on which the instruction problems were based, and the post-instruction tasks used to assess children's understanding of the major measurement concepts.

In addition to their use in designing instruction, the task analyses also identified the developmental abilities which are logically required to complete the tasks. That is, the analyses specified the demands made by each instruction problem and each post-instruction task on the developmental abilities. As Resnick (1976) and Shulman (Note 13) have pointed out, attention to these psychological components of the task is a particularly important aspect of task analyses which are concerned with the intellectual processes involved in task solution.

In this study, the task analyses specified the demands made by each measurement task on the developmental abilities of length conservation and length transitivity. Although most concepts of linear measurement are related in a general way to conservation and transitivity, they are not equally dependent on these abilities. Piaget makes a distinction between logical-mathematical knowledge and physical knowledge, and a related distinction between operative learning and figurative learning (see Chapter II). These distinctions suggest that various learning tasks may place different types of demands on children's developmental abilities. The analysis of the measurement tasks which was used in this study showed that some of the tasks were heavily dependent on logical-mathematical knowledge while others were based primarily on physical knowledge and required application of a measurement technique. Tasks which require operational learning or logical-mathematical knowledge may be more dependent on developmental capabilities than those requiring figurative learning or physical knowledge.

Ideally, a final analysis should be carried out to specify the information processing or M-space demands of each task. However, as alluded to in Chapter I, the application of M-space to instructional contexts is not sufficiently advanced to prescribe methods for analyzing complex mathematical tasks in terms of their M-space demands. Recent work by Kintsch and van Dijk (1978) in the area of verbal learning suggests that the information processing demands of instructional tasks can be specified in terms of task content variables. However, similar analyses have not yet been carried out for school mathematics tasks. Consequently, in this study M-space was treated in a more global way as a general integration capacity.

Research Hypotheses

As outlined in Chapter I, the major questions of interest concern developmental group differences in learning measurement concepts and descriptions or characterizations of the measurement strategies used by children within each developmental group. Some of these questions lend themselves to statistical analysis; others must be handled descriptively. The following section specifies the various hypotheses and provides a rationale for their inclusion.

Statistical questions. The analyses discussed in the preceding sections generated predictions about the effect of the developmental abilities on children's performance over the course of instruction. The predictions of primary interest concern differences in performance on certain measurement tasks between operational and preoperational children, and between low M-space and high M-space children. However, before these hypotheses could be tested, a check was needed on the stability of the sample in terms of the developmental factors.

The following hypotheses are presented as substantive questions of interest rather than as statistical hypotheses about observed scores.

> Hypothesis 1. Children's performance on length conservation/length transitivity and backward digit span will remain stable over short-term instruction on linear measurement.

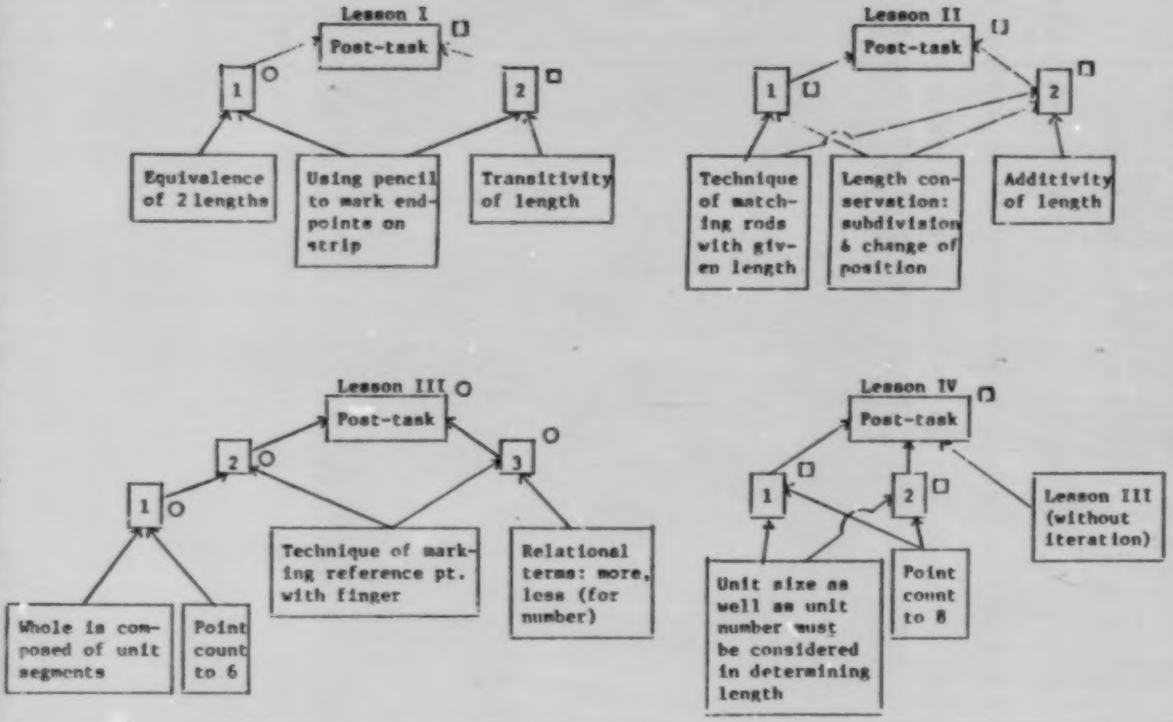
All of the questions which deal with the developmental notions of conservation, transitivity, and M-space assume that over a short time period they represent relatively stable constructs. It was assumed in this study that performance on these measures would remain stable over the brief instructional period. A check on this assumption constituted the first research question since a rejection of this hypothesis would alter the interpretation of most results.

Hypothesis 2

a) Operational children will perform better than preoperational children on the instruction problems and post-instruction tasks which depend on logical-mathematical knowledge.

b) Operational and preoperational children will not perform differently on the instruction problems and post-instruction tasks which require only physical knowledge or measurement technique.

Based on the analysis which specified the demands made by each instruction problem and post-instruction task on the developmental abilities of conservation and transitivity, the tasks were labeled as either logical-based or technique-based tasks. Figure 2 indicates these designations and specifies the prerequisites for each problem or task which were used to determine its classification. It was hypothesized that those problems or tasks which are based on logical-mathematical knowledge would show the effects of logical reasoning ability. Therefore it was on this set of tasks that differences between high reasoning-ability


Lesson I
Post-task
1
2
Equivalence of 2 lengths
Using pencil to mark end-points on strip
Transitivity of length
Lesson II
Post-task
1
2
Technique of matching rods with given length
Length conservation: subdivision & change of position
Additivity of length
Lesson III
Post-task
1
2
3
Whole is composed of unit segments
Point count to 6
Technique of marking reference pt. with finger
Relational terms: more, less (for number)
Lesson IV
Post-task
1
2
Unit size as well as unit number must be considered in determining length
Point count to 8
Lesson III (without iteration)

Numbers represent the instruction problems.

□:Tasks requiring logical-mathematical knowledge.

○:Tasks depending primarily on physical knowledge or specific measurement techniques.

Figure 2. Logical Analysis of Post-Instruction Tasks and Instruction Problems

and low reasoning-ability children were predicted to occur.

Hypothesis 3

a) High M-space children will perform better than low M-space children on the post-instruction tasks.

b) High M-space and low M-space children will not perform differently on the instruction problems.

Differences between low M-space and high M-space children in measurement task performance were hypothesized to be evident on the post-instruction tasks but not on the instruction problems. Each instruction problem focused on one prerequisite skill or concept, while the post-instruction task required the integration of these skills. Since M-space represents working short-term memory, i.e., an information integration capacity, its effect should be most evident on those tasks requiring the integration of newly learned skills or concepts. Case (1975) points out that M-space capacity often comes into play at the point where children have learned all the prerequisite skills and must integrate them to complete the superordinate task. Consequently, differences between the M-space groups were predicted on the set of post-instruction tasks but not on the set of instruction problems.

Hypothesis 4

a) Operational children will recognize and resolve conflict to a greater extent than preoperational children.

b) High M-space children will recognize and resolve conflict to a greater extent than low M-space children.

This represents the first of two hypotheses which consider the mediating role of cognitive conflict between the developmental abilities and performance over instruction. According to Piaget, a certain developmental level must be reached before the child is able to recognize the conflict generated by applying different strategies to solve the same task, and resolve this conflict by identifying the inadequacy of one strategy or the other. Therefore, operational children should engage in this behavior to a greater extent than preoperational children. In addition, since the construct of M-space is theoretically the functional equivalent of Piaget's structural notions, it was predicted that high M-space children would recognize and resolve conflict to a greater extent than low M-space children.

> Hypothesis 5. Recognition and resolution of conflict will relate positively to performance on the post-instruction tasks.

The second hypothesis which concerned the role of conflict resolution in learning measurement focuses directly on the relationship between recognition and resolution of conflict and subsequent performance on the measurement tasks. By switching the conflict variable from dependent to independent status this question considers the importance of the ability to recognize and resolve conflict on the learning of linear measurement concepts. The process of resolving cognitive conflict represents a central tenet of Piaget's position on the relationship between learning and development, and on the mechanism by which learning occurs. It was therefore hypothesized that children who recognize and resolve conflict during instruction would learn more about linear measurement than children who do not.

Descriptive questions. The statistically testable research questions in this study deal with between-group differences. These questions focus on comparisons of various developmental groups in terms of their mean performance on the measurement tasks. Several questions of interest still remained which could not be adequately handled by statistical tests. These questions were addressed using descriptive procedures.

Two major areas of interest provide the focus for the descriptive analyses. The first is the characterization of performance within particular developmental groups. Whereas the statistical analysis employed in this study considers differences between developmental groups and indicates whether the high developmental children learned more, on the average, than the low developmental children, the descriptive procedures were designed to characterize the absolute performance level of individual developmental groups. The question of interest pertains to the constraints which the lack of developmental abilities impose on learning measurement. Consequently, this descriptive analysis was directed toward the low developmental groups and the question was whether these children had learned the measurement concepts or skills. In other words, are certain developmental abilities required to learn about measurement; and if so, which abilities are required to learn which concepts or skills.

The second area of interest is a description of measurement strategies used by individual children or clusters of children on specific measurement tasks. An attempt was made to characterize the strategies used by the group of children in each cell of the developmental level matrix. This description contrasted the strategies used by, for example, high M-space operational children with those used by high M-space preoperational children on specific measurement tasks.

A description of the strategies used by children with different cognitive characteristics provides a way of re-analyzing the tasks from the child's point of view. The logical task analysis used in this study (see Appendix A) was based on an analysis from the adult's perspective. While this type of analysis is useful, it does not always match the child's analysis. Children may approach and solve a task in a way which does not correspond with a priori "logic" and consequently may succeed from a unique set of prerequisites. For instance, some measurement tasks logically involve the application of the transitivity principle. A comparison of the strategies used by children who reasoned transitively on the developmental task with those who did not indicates whether in fact transitivity is required to complete the task in question. In summary, a description of the strategies used to solve particular measurement tasks begins to reveal the way in which children view these tasks, and the cognitive abilities which are required to solve them.

Background of Methodology

Interest has been expressed recently by educational and psychological researchers in methodologies which are sensitive to individual responses and individual change. Since the intent of the current study was to observe the strategies which individual children use to solve measurement tasks and to instruct children in one-to-one interview situations, these methodologies contributed to the design of the study.

A central feature of many methods which focus on the individual, rather than the group, has been referred to as the case study approach. Stake (1978) and MacDonald and Walker (1975) have pointed out the merits of the case study method in educational research. Case studies are designed to describe and characterize individuals, rather than groups. They argue that this approach can be used effectively when understanding, rather than proof, is desired. Because of their compatability with people's experiential understanding, case studies can be used to increase understanding of the phenomena in question. Shulman (Note 14) reiterated this point by noting the importance of attending to the idiosyncratic cases--the ones which do not fit the norm. Here again the concern is with understanding rather than statistical certainty. In the area of mathematics education research, Ginsburg (1976, 1977) has consistently argued for the use of clinical interviews to determine how children think about mathematical problems. That such interviews may reveal critical aspects of the child's thinking which do not show up on conventional group tests has been convincingly demonstrated by Erlwanger (1975).

Several non-traditional methodologies have been used to study the effects of instruction. Soviet psychologists (e.g., Menchinskaya, 1969) have employed longitudinal designs together with natural classroom settings and individual interviews to investigate the effects of various instructional approaches on the development of children's mathematical concepts (see Kantowski, Steffe, Lee, & Hatfield, Note 15). Recently, Piaget and associates (Inhelder et al., 1974) launched a series of training experiments designed to reveal in more detail the mechanisms of development. They suggest several guidelines for studying the effect of development on learning. Children's developmental status is assessed by their performance on developmental tasks logically related to the learning content. Each child is then followed over the course of instruction. Differences between children in their ability to benefit from instruction are related to their initial developmental levels. Wohlwill (1973) recognizes this as a viable procedure for studying the effects of one factor on the change in another.

The specific method used in this study was not identical to any of the methods used in the previously cited investigations. However, it does reflect the spirit of them all. The study was a form of teaching experiment, but unlike the Soviet experiments, many of the instruction variables were controlled and protocols were relatively standardized to increase the generalizability of results. Children were instructed individually following the principles of case study and clinical interview techniques. Differences between children in their ability to benefit from instruction were related to their initial developmental levels in line with the suggestion of Inhelder et al. (1974). In summary, the overriding concern in this study was with individuals' responses, with changes in these responses over instruction, and with the effect of development on these changes.

Procedures

The concern with inter-individual differences and intra-individual change guided the selection and implementation of procedures. Methods were chosen which would permit a detailed description of the performance of children of different developmental levels as they were learning about measurement. It was especially important to characterize the processes or strategies which children use to measure, and the way in which these strategies change over the course of instruction. The remainder of this chapter will detail the procedures which were used to elicit, record, describe, and analyze these strategies.

Sample selection. The sample consisted of 32 first-grade children drawn from three elementary schools in Madison, Wisconsin. Subjects were selected on the basis of two criteria: developmental level and measurement knowledge. A pretest was used to assess children's developmental status on the Piagetian and information processing variables, and to test their existing knowledge of the measurement concepts on which instruction would be provided. With respect to the Piagetian variables, children were classified as concrete operational, transitional, or preoperational based on their cumulative performance on tasks of length conservation and length transitivity. An equal number of operational and preoperational children were selected for the study. Operational children were those who succeeded on both tasks; preoperational were those who failed both tasks. Transitional children were excluded from the sample for two reasons. First, children who are in the transitional stage from preoperations to concrete operations exhibit rather unstable performance on concrete operational tasks. They are more likely to be influenced by slight changes in task format, perceptual cues, etc. Since some of the measurement tasks to be used during instruction necessarily incorporated concepts of conservation and transitivity, it was important to only include children who dealt with these concepts in a consistent way, i.e., who responded similarly to these concepts across changes in task format. Second, since this study represented an initial investigation in this area it was desirable to maximize the difference in developmental levels represented in the sample, i.e., to maximize the potentially differential effects of development on learning.

Information processing capacity, or M-space, was assessed using a backward digit span test. Previous work (Lawson, 1976; Carpenter & Hiebert, Note 5) shows that about an equal number of first-grade children fall into one of two categories: those who succeed with a 3-digit series and those who do not. Very few children fail with two digits or pass with more than three digits. The 2- and 3-digit categories will be referred to here as low and high M-space. The final sample included an equal number of high and low M-space children. Therefore, with respect to the developmental variables, the final sample consisted of 32 children with eight in each cell of a 2 X 2 matrix of operational/preoperational by high/low M-space.

The second portion of the pretest assessed children's existing knowledge of linear measurement. This part of the pretest was constructed from the logical task analyses of the major measurement concepts (see Appendix A). The prerequisites identified through these analyses were of three types: high-level prerequisites which provided the focus for instruction; developmental abilities which were used to select the sample and formulate hypotheses of where during instruction developmental differences would be found; and, low-level prerequisities which were used to screen subjects from the sample. The first part of the measurement pretest focused on the lower-level prerequisite skills.

Children's performance on most of these prerequisites was assessed as a part of other items on the pretest. For example, the ability to make direct comparisons of lengths was assessed as part of the length transitivity task. Since instruction was not provided on these skills, children who did not have them were excluded from the sample. The sample therefore contained only children who possessed all of the (non-developmental) prerequisite skills upon which instruction would build.

The second part of the measurement pretest consisted of forms of the post-instruction tasks which were given at the conclusion of each lesson. These tasks were designed to assess children's understanding of the major measurement concepts on which instruction was given. Only children who were unsuccessful on all of these tasks were included in the sample. This selection procedure ensured that all children entered instruction with a similar level of measurement expertise. A description of all the items contained in the pretest, along with the protocols for their administration and the scoring criteria, are given in Appendix B.

A total of 143 children were pretested in order to identify 32 children who filled the 2 X 2 developmental level matrix and who satisfied the measurement knowledge criteria. Appropriate procedures were used for obtaining parent and school permission for subject participation. All subjects for whom approval had been received were pretested. The pretest items were given to all subjects in the same order: counting to determine the cardinality of a set, length transitivity, length conservation, forms of the four post-instruction tasks in the same order as the instruction lessons, and backward digit span. In order to reduce testing time and make minimal demands on students and teachers, pretesting with a particular subject was terminated as soon as the subject was eliminated from the final sample according to the criteria described above. Consequently, many of the subjects received only some of the pretest items.

Pretesting began with 35 children at School A. Twelve of these children were included in the sample; one of them was high Piagetian--high M-space, four were low Piagetian--high M-space, and seven were low Piagetian--low M-space. Of the 70 children pretested at School B, 14 were included in the sample. Three of these were high Piagetian--

high M-space, six were high Piagetian--low M-space, four were low Piagetian--high M-space, and one was low Piagetian--low M-space. Pretesting concluded with 38 children at School C. Six of these were included in the sample; four were high Piagetian--high M-space and two were high Piagetian--low M-space. A summary of the pretest performance of all 143 potential subjects is given in Appendix H.

The sample size of 32 was selected primarily on the basis of external constraints such as testing and instruction time, availability of initial pool of subjects, etc. Thirty-two was considered the maximum number given these constraints. A power analysis was carried out to determine the probability of detecting between-group differences of a specified magnitude with this size sample. With $\alpha = .05$, $N = 32$ (8 subjects per cell), and $\eta^2 = .20$, the power for two-way analysis of variance tests is .76 (Cohen, 1977). This is a reasonable level of power.

A word of explanation is in order about η^2, a measure of the magnitude of differences which are hypothesized to be present in the population. The coefficient η is interpreted as a partial correlation coefficient and η^2 as the proportion of variance accounted for by one of the factors with the other factor and interaction held constant or partialled out. In this study it is assumed that 20% ($\eta^2 = .20$) of the variance in performance on the linear measurement tasks can be accounted for by population membership in either of the developmental groups (Piagetian or M-space). The position taken here is that, although $\eta^2 = .20$ is a comparatively optimistic expectation, if the between-group differences are less than substantial in this relatively controlled instruction situation, then the developmental variables have questionable educational value as predictors of children's ability to benefit from classroom instruction. Relatively large statistical differences must exist before they can be considered *educationally* significant.

Instruction and Assessment. The data collection procedures consisted of a pretest for purposes of sample selection, an instructional sequence consisting of instruction and assessment, and a posttest to [illegible]re developmental change. It should be noted that the procedures which were used to assess learning during instruction were built in as part of the instructional sequence. For each subject, all testing and instruction was conducted within a three-week period.

After the 32 subjects were selected according to the criteria specified earlier, they were presented with four instruction lessons. The lessons were given individually, and each subject received no more than one lesson per day over the course of 5-6 school days. Each lesson focused on one fundamental principle of linear measurement as described previously.

All of the instruction lessons were designed to follow a similar progression. From a developmental perspective children learn to represent the lengths of objects. They are then able to use this representation as an intermediary measure to compare and order two objects on the basis of length, and then to construct a second length equal to a first. In general, the instruction lessons followed this developmental pattern.

Each lesson consisted of a series of instruction problems which focused on the primary prerequisite skills or concepts required to successfully complete the post-instruction task. These problems proceeded from a simple construction or representation of length to a compare and order situation. Each problem required the child to measure a length in some way. The experimenter then measured the same length in a different way. If the child had measured incorrectly, the experimenter measured correctly; if the child had measured correctly, the experimenter measured incorrectly in a predetermined way. The experimenter then asked the child to explain the reason for the different results. The purpose of this procedure was to introduce cognitive conflict. After the child was given opportunity to resolve this conflict, the experimenter verbalized the measurement principle involved. That is, the experimenter stated the principle in appropriate language so as to make available to the child the information needed to complete the task. Opportunity was then given for the child to apply this information and practice the skill on a task which was similar in structure but different in form to the initial task. This task permitted an assessment of the child's knowledge of that prerequisite skill or concept after instruction. No feedback was given on this practice trial. The experimenter then moved to the next instruction problem. After all instruction problems were completed the post-instruction task was presented. This task was designed to assess the child's understanding of the major measurement principle of that lesson. It required an integration of the prerequisite skills and a synthesis of the representation and the compare and order operations covered during instruction. No feedback was given on this task. A complete description of the instruction problems and the post-instruction tasks for all lessons is given in Appendix C. The lesson protocols are presented in Appendix D.

The instruction procedure used in this study can be further characterized by identifying its five salient features. First, instruction was provided in a one-to-one setting. This format maximized learning opportunity by encouraging student-teacher interaction, by permitting individualized feedback from the teacher, and by ensuring a high level of engaged time for the learner. The importance of engaged time in learning situations has been recently demonstrated (Berliner, Note 16). Individually administered instruction also made possible the recording of individual responses during instruction. It allowed detailed observation of the processes children used to measure and the ways in which they responded to instruction.

The second feature of instruction was that children were required to carry out actual measurements on a series of instructional problems. This activity oriented approach encouraged on-task or engaged behavior. It is also consonant with viewing the learner as actively engaged in constructing knowledge, a perspective taken by many psychologists and educators (see e.g., Elkind, 1976; Lesh, 1973; Osborne, 1976; Wittrock, 1978; Romberg & Harvey, Note 17). Furthermore, requiring children to overtly measure permitted observation of the strategies children used to measure. This in turn revealed children's conceptions of linear measurement. Requiring children to actively manipulate objects in conjunction with questions about these manipulations has been established as a fruitful way to investigate children's thinking. In fact it is the basis for Piaget's methode clinique.

A third feature of instruction was the attention given to the logical prerequisites of the instruction objectives. Designing instruction which accounted for the prerequisite skills contributed to the effectiveness of instruction and permitted an investigation of the factors responsible for variation in learning performance. For example, the post-instruction task, which assessed the effect of the preceding instruction, represented an integration of the highest-level prerequisites. Failure on this task could be attributed to an incomplete mastery of one or more of these skills, or to a failure to integrate them. Children's performance on each of the instruction problems indicated their level of mastery of each prerequisite. Assuming an adequate task analysis, mastery of all prerequisites and failure on the terminal (post-instruction) task suggested a limited integration capacity (M-space). Therefore, a less than successful performance on the post-instruction task could be traced to an incomplete mastery of a prerequisite or to an inability to integrate these individual skills (see Figure 2).

A fourth feature of instruction was the form of student-teacher (subject-experimenter) interaction which occurred in each instruction problem. The most important aspect of this interaction was the introduction of cognitive conflict by the experimenter. A method of generating conflict in measurement situations has been illustrated by Inhelder et al. (1974). Similar procedures were used in this study.

Situations of cognitive conflict presumably serve to promote operational learning (Furth, 1970; Lovell, 1966; Smock, 1976; Hooper & DeFrain, Note 1). Such situations place the learner in a position which calls for a rethinking or reorganization of existing conceptions. From a Piagetian perspective, the recognition and resolution of cognitive conflict is a form of equilibration, the mechanism postulated to govern operational learning and development (Piaget, 1971a).

The fifth characteristic of instruction was its completeness with respect to providing the information necessary to carry out the required measurement and providing opportunity for practice. After children were given the opportunity to resolve the conflict introduced in the instruction problem, the experimenter verbalized the measurement principle involved. The children were then given a problem on which they could practice their newly learned skill. In addition to promoting acquisition of that skill, the practice task permitted an assessment of the child's mastery of that prerequisite skill or concept after instruction. It therefore provided essential information on the role of that particular prerequisite in the child's performance on the post-instruction task.

Several days after instruction was completed, each subject was given a cognitive task posttest. This consisted of a readministration of the cognitive developmental tasks which were given in the pretest: length conservation, length transitivity, and backward digit span. Theoretically, performance should remain stable on these tasks over the brief instruction period. The posttest was given to check on this assumption.

All testing and instruction was conducted by the experimenter.

A trained observer was present during the instruction lessons to record children's measurement strategies and their responses to the cognitive conflict situations.

Coding Responses

The initial problem in analyzing data which consists of descriptions of children's solution processes or strategies is one of coding. The overwhelming amount of information must be organized and translated into some manageable form. The dilemma is that some reduction and scaling of the data is necessary in order to apply available statistical procedures and to abstract general patterns or characteristics from the myriad of individual responses; but an over-reduction of the data may lose important information about individual responses and specific strategies. The analysis to be used here deals with this problem by reporting the results at two levels. Based on pilot studies and a logical analysis of the tasks, lists of strategies by which children could complete the instruction problems and the post-instruction tasks were constructed for each lesson (see Appendix E). These strategies were used to classify the processes children used to measure. The purpose of this level of coding was to retain all non-trivial information on children's measurement processes. This data was used as the basis for the descriptive analysis.

Some scaling or reduction of the data was required for the statistical analysis. While each of the lists of measurement strategies show a general progression from perceptually-bound to conceptually-based strategies, a complete scaling of all strategies within each list was not possible. For example, there are usually several incorrect strategies which evidence a similar level of understanding of the measurement concept in question. It was possible, however, to classify all strategies into one of three, ordinal categories. The lowest level category included those strategies based on perceptual judgments and/or showing no resemblance to appropriate measurement techniques. The second category consisted of those strategies which were partially correct, i.e., which evidenced some understanding of the measurement principle in question, but which for some reason did not achieve an accurate result. The inaccurate results were often due to a deficient measuring technique. The final category was limited to those strategies which evidenced an understanding of the measurement principle and which yielded an accurate result. Using these criteria, children's measurement strategies were scored 0, 1, or 2 on each problem and task. (See Appendix E for the specific scoring criteria for each problem and task.) These data were used for the statistical analyses.

In addition to coding children's measurement strategies, some measure was needed of children's ability to recognize and resolve conflict during instruction. Presented with a conflict situation, children can respond in one of three ways: they may not recognize the conflict and therefore see no need to resolve the different results; they may recognize the conflict but not be able to resolve it; or, they may recognize the conflict and resolve it by explaining or demonstrating the reason for the different results. Using the more specific scoring criteria given in Appendix E, children's responses in the conflict situation were classified into one of these three categories and scored 0, 1, or 2.

Taken together, the scores for measurement task performance and the conflict scores generated a vector which characterized, in a quantitative way, each subject's performance throughout instruction. Figure 3 identifies the instruction problems and the post-instruction tasks, and depicts the vector of scores which was constructed for each subject. All statistical analyses were based on these data.

Observer Agreement. As indicated previously, observers were trained to record and score children's measurement strategies and their responses to the conflict situations. Two observers were trained using videotapes of first-grade children receiving the instruction lessons from the experimenter. In line with the recommendations of Frick and Semmel (1978), observer agreement

LESSON I

Instruction Problems

1

Construct a continuous representation of the length of a given object

2

Compare and order two lengths using an intermediary continuous representation

Post-instruction task

Construct a 2nd length equal to a 1st using an intermediary continuous representation

LESSON II

Instruction Problems

1

Construct a 2nd straight length equal to a 1st but beginning at a different point (using discrete linear segments)

2

Construct a 2nd length equal to the sum of two separate linear segments

Post-instruction task

Construct a 2nd straight path equal to a 1st polygonal path using discrete linear segments

One vector will be constructed for each subject; scores are 0, 1, or 2 for each measure

Lesson I, Problem 1				Post-task					
I.1 RC	I.1 PT	I.2 RC	I.2 PT	I.P	II.1 RC	II.1 PT	II.2 RC	II.2 PT	II.P
Recognition and resolution of conflict(RC)	Practice task(PT)								

Figure 3. Instruction Lessons and Vector of Scored Responses

LESSON III

Instruction Problems

1

Represent a length numerically by measuring it with a collection of units

2

Represent a length numerically by measuring it with a single unit (unit iteration)

3

Compare and order two lengths by measuring with a single unit and comparing measures

Post-instruction task

Construct a 2nd length equal to a 1st using a single unit to measure (unit iteration)

III.1 RC **III.1 PT** **III.2 RC** **III.2 PT** **III.3 RC** **III.3 PT** **III.P**

LESSON IV

Instruction Problems

1

Construct a 2nd straight length equal to a 1st using different size units

2

Compare and order 2 lengths measured with different size units

Post-instruction task

Construct a 2nd straight path equal to a 1st polygonal path using different size units

IV.1 RC **IV.1 PT** **IV.2 RC** **IV.2 PT** **IV.P**

(Figure 3 continued)

was computed both prior to, and during, data collection. For purposes of computing observer agreement prior to data collection, an additional tape was shown of one child receiving all four instruction lessons. Since the basic unit of analysis was the score assigned to each measurement strategy and each response to the conflict situation, the percentage of observer agreement was computed from these scores. Using the experimenter as the criterion, Observer A agreed on 91.7% (11/12) of the codeable measurement scores and 75% (6/8) of the codeable conflict scores. Observer B agreed on 100% (12/12) of the codeable measurement scores and 87.5% (7/8) of the codeable conflict scores. Due to problems associated with video-taping, one conflict response and one measurement response were unclear and were not scored.

To check observer agreement during instruction, the experimenter scored the responses of several children, selected at random, for each instruction lesson. To compute percentage of observer agreement, three protocols for each lesson were randomly selected from this set. Since only one observer was present for a particular lesson, the agreement for Observer A could only be computed for Lessons 1 and 2, and that for Observer B for Lessons 3 and 4. Using the experimenter as the criterion, Observer A agreed on 94.2% (17/18) of the measurement scores and 83.3% (10/12) of the conflict scores. Observer B agreed on 100% (21/21) of the measurement scores and

93.3% (14/15) of the conflict scores.

Analysis Procedures

Statistical analysis. Information was collected from each child in this study on 24 measures. One of these was a collective measure of Piagetian operational level with respect to length; one was a measure of M-space or information processing capacity; nine were measures of ability to recognize and resolve conflict; and the remaining 13 were the linear measurement assessment tasks presented during instruction. Some information on the relations between these variables was available from logical analyses. For example, a subset of the linear measurement tasks were closely related from a logical standpoint since they required logical-mathematical knowledge for solution. The complementary subset of tasks were also related since they depended primarily on physical knowledge or measurement technique (see Figure 2). It was this kind of information that was utilized in the statistical analysis. Differences between the Piagetian-level groups were investigated using a partitioning of tasks along the dimension logically related to the Piagetian variables. A similar logic held for investigating M-space between-group differences.

The statistical procedures which were used will be described with respect to the research hypotheses presented earlier. Hypothesis 1 (see p. 107) was tested using the Pearson goodness-of-fit test (Marascuilo & McSweeney, 1977). This is a nonparametric test which measures the agreement between an obtained distribution and a theoretical or expected distribution. Theoretically, performance on the Piagetian and M-space tasks should remain relatively stable over a short time period. Consequently it was expected that the sample would be equally distributed over the cells of the 2 X 2 developmental level matrix on the posttest.

Analysis of variance procedures were used for hypotheses 2 and 3 (see pages 108 and 109) with Piagetian operational level and M-space serving as the independent variables. These hypotheses focused on differences between the developmental groups in performance on the measurement tasks. Since the logical analyses indicated the particular tasks on which these differences were expected to occur, the scores were aggregated as follows. With respect to the Piagetian factor, scores on the logical-mathematical tasks were summed to create one dependent variable (hypothesis 2a) and scores on the technique tasks were summed to create a second dependent variable (hypothesis 2b). The tasks were partitioned in a different way to investigate the effect of M-space. The sum of scores on the post-instruction or integration tasks formed one dependent variable (hypothesis 3a) and the sum of scores on the instruction problems or individual skill tasks formed a second dependent variable (hypothesis 3b). Reliability coefficients were computed to measure the internal consistency of each set of scores.

It should be noted that the partitioning of tasks for hypotheses 2 and 3 represented two different aggregations of the same set of scores, hence tests of the hypotheses are not statistically independent. This is reasonable, however, since all the tasks were analyzed from two different perspectives: One analysis involved the logical-mathematical component or the demands placed on logical reasoning abilities, and a second analysis involved an integration component or the demands placed on M-space. Since all the tasks were represented in both partitionings, there was potential for an interaction between the developmental factors. For example, three of the four post-instruction tasks were logical-mathematical tasks. It may have been that children needed to be developmentally advanced along both dimensions to successfully complete these tasks. Two-way analysis of variance procedures were used to test these possible interaction effects. However, for hypothesis 2, primary interest was on the main effect of the Piagetian between-group difference, and, for hypothesis 3, the M-space between-group difference.

Hypothesis 4 (see p. 110) was also tested using analysis of variance procedures. The independent variables were Piagetian operational level and M-space capacity. The dependent variable was the sum of scores on the recognition and resolution of conflict measures.

Hypothesis 5 (see p. 11) was treated as a regression problem with recognition and resolution of conflict serving as the independent variable and post-instruction task performance forming the dependent variable. Both variables were measured by simply summing scores through instruction.

Descriptive analysis. The primary objective of the descriptive analysis was to characterize performance within a particular developmental group. Contingency tables were prepared for each of the measurement tasks showing the number of children in each developmental group who obtained a particular score on each task. This information was used to determine whether a given task is accessible to children who lack the logical reasoning abilities or who have a small information processing capacity.

An attempt was also made to characterize the measuring strategies used by children in each cell of the developmental level matrix. The focus of this descriptive analysis was on the types of strategies used by children possessing certain cognitive characteristics. For example, some measuring strategies appear to involve the application of logical reasoning abilities (i.e., conservation and transitivity). The question is whether children who do not conserve or reason transitively employ these kinds of strategies. This information may provide some insight into the way in which these developmental abilities affect children's measuring performance.

A final question of interest which was addressed using descriptive procedures is whether information processing capacity affects children's ability to integrate individually mastered skills in solving a superordinate task. In this study, the instruction problems represented individual skills which were logically required to complete the post-instruction task. A frequency count of the number of children in each developmental group who mastered the prerequisite skills but failed the post-instruction task indicates whether a high capacity was necessary for, or facilitated, the integration of separate skills.

Chapter V

RESULTS

The results of the study will be presented for each of the hypotheses given on pp. 107-111. These hypotheses are restated here in null hypothesis form. In most cases statistical tests were used to assess differences between the developmental groups on various aggregations of measurement and conflict scores. Performance on individual measurement tasks will also be described both quantitatively and qualitatively. This analysis focuses on a characterization of performance within each cell of the developmental level matrix and descriptive comparisons between cells on particular tasks.

Stability on Developmental Variables

> Hypothesis 1. Children's performance on length conservation/length transitivity and backward digit span will remain stable over short-term instruction on linear measurement.
>
> Null Hypothesis 1. The observed frequencies on the posttest will not differ significantly from the expected frequencies of equal numbers in each cell of the developmental level matrix.

Performance on the posttest length conservation and length transitivity tasks showed that four children had moved from preoperational to transitional and two children had moved from operational to transitional. With respect to performance on the backward digit span task, one child had moved from low M-space to high M-space and three children had moved from high M-space to low M-space. Therefore, compared to pretest performance, about 81% of the subjects gave identical responses on the Piagetian posttest and about 88% of the subjects gave similar responses on the backward digit span posttest.

In order to conduct a liberal statistical test to detect significant shifts in the sample, changes on the Piagetian tasks were treated as changes from one level to the other rather than as changes to transitional responses. Using this modification, Table 1 shows the expected and observed distributions on the posttest developmental tasks.

Comparing these two distributions, the Pearson χ^2 statistic for goodness-of-fit is 2.75. This is well below the critical value of 7.81 (α=.05, df=3). Therefore the hypothesis of a nonsignificant systematic shift in the sample was not rejected. The remaining analyses were run using the initial classification of subjects.

Effects of Developmental Variables on Measurement Performance

Logical Reasoning Ability. The first partitioning of the linear measurement tasks was in terms of the types of knowledge needed to complete them, i.e., the demands they placed on the logical reasoning abilities. One aggregation consisted of logical-mathematical tasks or tasks which logically required length conservation or length transitivity for solution. The remaining tasks depended primarily on a specific measurement technique for solution.

> Hypothesis 2a. Operational children will perform better than preoperational children on the instruction problems and post-instruction tasks which depend on logical-mathematical

Table 1

Posttest Performance on the Developmental Tasks

Observed Frequencies

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	9	9
	Low	4	10

Expected Frequencies

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	8	8
	Low	8	8

Table entries represent number of subjects in each category.

knowledge.

> Null Hypothesis 2a. There is no significant difference in the mean scores of the two Piagetian level groups on the logical-mathematical tasks.

Means and standard deviations for each developmental group on the set of logical-mathematical tasks are shown in Table 2. Analysis of variance on these scores is summarized in Table 3.

The two Piagetian level groups differed significantly ($\alpha = .05$) on this set of tasks and the null hypothesis is rejected. The logical reasoning ability factor accounted for 23.0% of the variance in children's performance. The low developmental children had a mean total score of 9.13 (out of 16) while the high developmental children had a mean total score of 12.81. The interaction between the developmental factors was not significant. The Cronbach alpha reliability coefficient for this set of eight tasks was .75.

> Hypothesis 2b. Operational and preoperational children will not perform differently on the instruction problems and post-instruction tasks which require only physical knowledge or measurement technique.

> Null Hypothesis 2b. There is no significant difference in the mean scores of the two Piagetian level groups on the technique tasks.

The prediction in the substantive hypothesis indicates the expectation that the null hypothesis will not be rejected.

Table 4 presents means and standard deviations and Table 5 summarizes the analysis of variance on the aggregation of technique-based tasks.

Table 2

Means and Standard Deviations--

Logical-Mathematical Tasks

		Information Processing Capacity		
		High	Low	
Logical Reasoning Ability	High	$\bar{X}=13.38$ SD=2.20	$\bar{X}=12.25$ SD=3.37	$\bar{X}=12.81$ SD=2.81
	Low	$\bar{X}=9.13$ SD=4.94	$\bar{X}=9.13$ SD=3.23	$\bar{X}=9.13$ SD=4.03
		$\bar{X}=11.25$ SD=4.30	$\bar{X}=10.69$ SD=3.57	

Maximum score = 16

Table 3

ANOVA--Logical-Mathematical Tasks

Source of Variation	df	MS	F	$p<$	η^2*
Logical reasoning ability (A)	1	108.781	8.529	.007	.230
Information processing capacity (B)	1	2.531	.198	.659	.005
A x B	1	2.531	.198	.659	
Residual	28	12.754			

*η^2 indicates the proportion of variation in performance on this set of tasks explained by each factor.

Table 4

Means and Standard Deviations--

Technique Tasks

		Information Processing Capacity		
		High	Low	
Logical Reasoning Ability	High	$\bar{X}$=6.88 SD=1.25	$\bar{X}$=7.63 SD=2.33	$\bar{X}$=7.25 SD=1.84
	Low	$\bar{X}$=5.88 SD=1.36	$\bar{X}$=7.75 SD=1.75	$\bar{X}$=6.81 SD=1.80
		$\bar{X}$=6.37 SD=1.36	$\bar{X}$=7.69 SD=1.99	

Maximum score = 10

Table 5

ANOVA—Technique Tasks

Source of Variation	df	MS	F	p<	η^2*
Logical reasoning ability (A)	1	1.531	.516	.479	.014
Information processing capacity (B)	1	13.781	4.642	.040	.137
A x B	1	2.531	.853	.364	
Residual	28	2.969			

*η^2 indicates the proportion of variation in performance on this set of tasks explained by each factor.

The Piagetian operational level groups did not perform significantly different on this set of tasks. The variation in performance on these tasks is therefore accounted for by factors other than logical reasoning ability. Although no predictions were advanced with regard to the information processing factor on these tasks, the main effect for information processing capacity was significant at the .05 level. This factor accounted for 13.7% of the variance in children's performance, with the low developmental children performing better ($\overline{X}$=7.69 out of 10) than the high developmental children ($\overline{X}$=6.37). The interaction between the developmental factors was not significant. Cronbach's alpha on this set of five tasks was .67.

Information processing capacity. The second partitioning of measurement tasks was based on the demands which they were expected to place on information-processing capacity (M-space). Since the post-instruction tasks required an integration of the separate skills taught during instruction, the aggregation of these scores presumably represented a high M-space demand score. In contrast, the instruction problems focused on an individual concept or skill and the aggregation of these scores represented a low M-space demand score.

Hypothesis 3a. High M-space children will perform better than low M-space children on the post-instruction tasks.

Null Hypothesis 3a. There is no significant difference in the mean scores of the two M-space groups on the post-instruction tasks.

Means and standard deviations on the post-instruction tasks are presented in Table 6. Analysis of variance on these data is summarized in Table 7.

Contrary to prediction, the effect of information processing capacity on children's ability to master these tasks was clearly nonsignificant and the null hypothesis is not rejected. The low and high developmental groups had identical mean scores of 3.94 (out of 8). The logical reasoning ability factor was significant, however, accounting for 31% of the variance in children's performance. The preoperational children had a mean score of 2.88 while the operational children had a mean score of 5.00. It must be remembered that three of the four post-instruction tasks were also classified as logical-mathematical tasks. The interaction between the developmental factors was not significant. Cronbach's alpha for this set of four tasks was .22.

> Hypothesis 3b. High M-space and low M-space children will not perform differently on the instruction problems.
>
> Null Hypothesis 3b. There is no significant difference in the mean scores of the two M-space groups on the instruction problems.

As in Hypothesis 2b, the prediction here is that the null hypothesis will not be rejected.

Means and standard deviations for the set of instruction problems are shown in Table 8. Analysis of variance on these scores is summarized in Table 9.

The information processing groups did not differ significantly

Table 6

Means and Standard Deviations--

Post-Instruction Tasks

		Information Processing Capacity		
		High	Low	
Logical Reasoning Ability	High	$\bar{X}=4.75$ SD=1.67	$\bar{X}=5.25$ SD=1.98	$\bar{X}=5.00$ SD=1.79
	Low	$\bar{X}=3.13$ SD=1.81	$\bar{X}=2.63$ SD=1.06	$\bar{X}=2.88$ SD=1.45
		$\bar{X}=3.94$ SD=1.88	$\bar{X}=3.94$ SD=2.05	

Maximum score = 8

Table 7

ANOVA--Post-Instruction Tasks

Source of Variation	df	MS	F	p<	η^2*
Logical reasoning ability (A)	1	36.125	13.010	.001	.314
Information processing capacity (B)	1	.000	.000	1.000	.000
A x B	1	2.000	.720	.403	
Residual	28	2.777			

*η^2 indicates the proportion of variation in performance on this set of tasks explained by each factor.

Table 8

Means and Standard Deviations--

Instruction Problems

Information Processing Capacity

Logical Reasoning Ability		High	Low	
	High	$\bar{X}=15.50$ SD=1.41	$\bar{X}=14.63$ SD=3.74	$\bar{X}=15.06$ SD=2.77
	Low	$\bar{X}=11.88$ SD=4.32	$\bar{X}=14.25$ SD=2.25	$\bar{X}=13.06$ SD=3.55
		$\bar{X}=13.68$ SD=3.63	$\bar{X}=14.44$ SD=2.99	

Maximum score = 18

Table 9

ANOVA--Instruction Problems

Source of Variation	df	MS	F	p<	η^2*
Logical reasoning ability (A)	1	32.000	3.220	.084	.096
Information processing capacity (B)	1	4.500	.453	.507	.014
A x B	1	21.125	2.126	.156	
Residual	28	9.937			

*η^2 indicates the proportion of variation in performance on this set of tasks explained by each factor.

in their performance on these tasks. The interaction of the developmental factors was also nonsignificant. Cronbach's alpha for this set of nine tasks was .75.

Effects of Developmental Variables on Recognition and Resolution of Conflict

Hypothesis 4

a. Operational children will recognize and resolve conflict to a greater extent than preoperational children.
b. High M-space children will recognize and resolve conflict to a greater extent than low M-space children.

Null Hypothesis 4

a. There is no significant difference in the mean scores of the two Piagetian level groups on the recognition and resolution of conflict.
b. There is no significant difference in the mean scores of the two M-space groups on the recognition and resolution of conflict.

A score representing children's ability to deal with cognitive conflict was obtained by summing the nine recognition and resolution of conflict scores. Table 10 displays the means and standard deviations on these scores. Table 11 summarizes the analysis of variance on this dependent measure.

Neither the main effect nor the interaction effect were statistically significant (α=.05). Therefore, neither of the null hypotheses are rejected.

Relationship Between Recognition and Resolution of Conflict and Measurement Performance

Hypothesis 5. Recognition and resolution of conflict will relate positively to performance on the post-instruction tasks.

Table 10

Means and Standard Deviations--

Recognition and Resolution of Conflict

		Information Processing Capacity		
		High	Low	
Logical Reasoning Ability	High	$\bar{X}=16.00$ SD=1.60	$\bar{X}=15.88$ SD=2.80	$\bar{X}=15.94$ SD=2.21
	Low	$\bar{X}=12.25$ SD=3.92	$\bar{X}=15.50$ SD=3.66	$\bar{X}=13.88$ SD=4.03
		$\bar{X}=14.12$ SD=3.48	$\bar{X}=15.69$ SD=1.66	

Maximum Score = 18

Table 11

ANOVA--Recognition and Resolution of Conflict

Source of Variation	df	MS	F	p<	η^2*
Logical reasoning ability (A)	1	34.031	3.473	.073	.096
Information processing capacity (B)	1	19.531	1.993	.169	.058
A x B	1	22.781	2.325	.139	
Residual	28	9.799			

*η^2 indicates the proportion of variation in performance on this set of tasks explained by each factor.

> Null Hypothesis 5. There is no significant correlation between the recognition and resolution of conflict and performance on the post-instruction tasks.

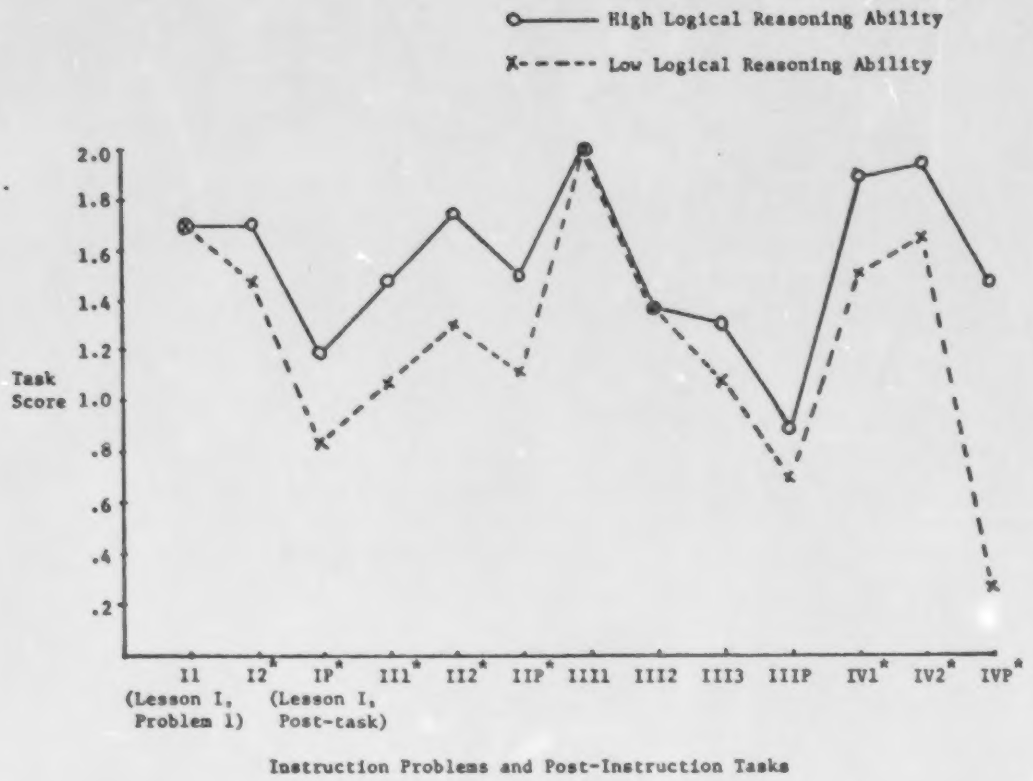
The conflict score was conceptually shifted from its dependent status in the previous analysis to independent status for the present analysis. Since the conflict situations were introduced as part of the instructional procedure, the post-instruction tasks assessed the effect of children's ability to recognize and resolve conflict on their measurement performance. Scores on the post-instruction tasks were summed to create a measurement performance score. A simple bivariate regression between these two variables yielded a correlation coefficient of $r=.455$ and a coefficient of determination $r^2=.207$. Therefore 20.7% of the variation in measurement performance is explained by linear regression on the conflict variable. An F ratio of 7.85 for the regression coefficient indicates that this linear association is statistically significant at the .01 level. The null hypothesis is rejected.

Description of Measurement Performance

Whereas the preceding statistical analyses considered between-group differences on groups of scores, the descriptive analysis focused on within-group performance on individual tasks and comparisons between developmental groups on particular tasks. The primary question of interest was whether the developmental abilities were required to master certain tasks, i.e., whether certain tasks were "inaccessible" to the low developmental children. An over-all picture of the performance of each developmental group can be obtained from the graphs in Figures 4 and 5.

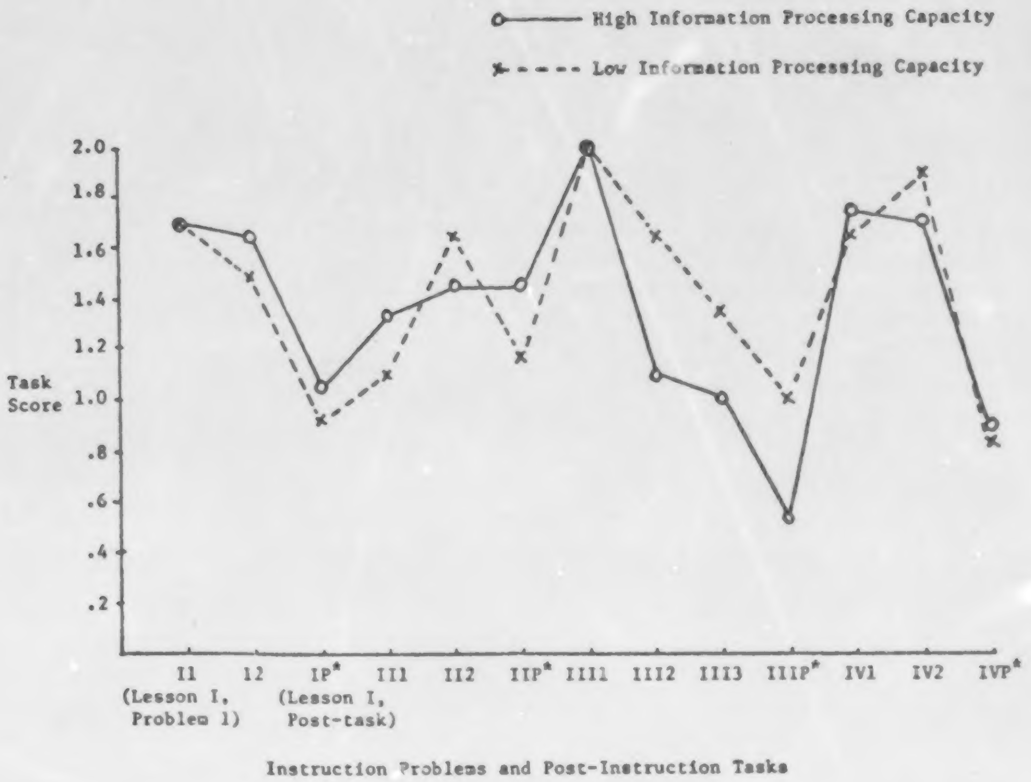
The graph in Figure 4 shows that the high developmental group had a higher mean score on all tasks except three. Consistent with the hypotheses, these were three of the five tasks on which group differences were not expected to occur. The mean scores of the low developmental group suggest that all of the tasks except for the post-instruction task in Lesson IV could be mastered by at least some of the preoperational children.. Contingency tables were computed for each task to check on this conjecture. These are shown in Appendix F. Patterns of scores within each task support the fact that, while not as many preoperational children achieved complete mastery of most of the tasks, the tasks were generally accessible to these low developmental children. The striking exception to this pattern was the post-instruction task in Lesson IV. Only two of the 16 preoperational children scored above 0 on this task. Both of these children were found to be in transition toward concrete operations on the posttest developmental tasks.

The graph in Figure 5 suggests no clear pattern. On some tasks, the high M-space children performed better than the low M-space children, while on others this ranking was reversed.



*Tasks on which group differences were predicted

Figure 4. Mean Performance on the Measurement Tasks by the Preoperational and Operational Groups



*Tasks on which group differences were predicted

Figure 5. Mean Performance on the Measurement Tasks by the High and Low M-space Groups

The mean scores suggest that all of the tasks were within the capabilities of at least some of the low M-space children. The contingency tables provided in Appendix F confirm this observation. The patterns of scores within each task, except one, are very similar for both developmental groups. The exception is the post-instruction task in Lesson III. None of the high M-space children were completely successful on this task, although nine children were partially successful.

Description of Measurement Strategies

The measuring strategies which children used to complete each task were recorded and assigned a number using the coding scheme detailed in Appendix E. In order to gain an understanding of how children viewed length and linear measurement, an attempt was made to characterize the strategies used by children in each developmental group on the post-instruction tasks. A complete list of the strategies used by each developmental group on each post-instruction task is given in Appendix G.

Lesson One. The post-instruction task in Lesson I required children to construct a second, moveable building equal in height to a first building using an intermediate representation (see Appendices C and D for complete descriptions of all tasks). Perceptual solutions were difficult since the second building was situated on a hill. The two most frequently used strategies were a correct strategy, in which the height of the first building was represented on the strip and then this representation was used to adjust the height of the second building, and an incorrect strategy in which the strip was laid horizontally and the second building was made "just as high in the sky." Both of these strategies were used by at least some children in each cell of the developmental level matrix. The incorrect strategy reveals a misconception of length which identifies equivalent lengths as the alignment of only one pair of endpoints. This misconception was evidenced by both high and low developmental children. It is possible, of course, that children simply did not understand the directions of the task. This problem was minimized, however, by using phrases "just as big" and "just as much room inside" rather than "just as high," and by demonstrating the meaning of these directions on a preceding problem.

From a logical perspective, transitive reasoning is involved in applying a correct measuring strategy in this task. The intermediate representation is used to indirectly compare the heights of the two buildings. However, eight of the 16 preoperational children used this kind of strategy. Even though these eight children failed the length transitivity task, they were able to apply a measuring strategy which would seem to involve this kind of reasoning.

In summary, it is difficult to distinguish between the developmental groups in terms of the strategies used on the Lesson

I post-instruction task. Both low and high developmental children used both unsuccessful and successful strategies. Consequently it is difficult to identify the role played by the developmental abilities in mastering this task.

Lesson Two. The post-instruction task in Lesson II required children to construct a straight path equal to a polygonal path using a collection of Cuisenaire rods. Two strategies, one correct and one incorrect, were used most frequently. The correct strategy involved matching a selection of Cuisenaire rods with the given path and then laying them out to make the required straight path. The incorrect strategy was a simple perceptual solution in which the straight path was made to "look just as long" as the polygonal path. The correct strategy was used by at least some of the children in all four developmental groups. The incorrect strategy was used by at least some children in all the developmental groups except for the high Piagetian--high M-space group. All of these eight children employed some kind of matching strategy.

A particular kind of error, which may signify an over-confidence in measuring ability, was committed only by high M-space children. This partially successful strategy involved matching rods with the given path but only in approximate and less than careful fashion.

As before, it is difficult to completely characterize the measuring strategies used within developmental groups, or to distinguish between developmental groups. In other words, it is difficult to identify the effect which the developmental abilities had on children's measuring strategies in this task. It is clear, however, that even low developmental children could apply appropriate strategies to solve the task.

Lesson Three. The post-instruction task in Lesson III required children to construct a second length equal to a first length by using unit iteration. A single Cuisenaire rod was provided for measuring; an accurate measuring technique had been demonstrated on the instruction problems.

The majority of children solved this task using some form of unit iteration. However, only six children were completely successful; the rest achieved an inaccurate solution due to some problem with the measuring technique. The most striking result was that all six of these successful children were low M-space children--not one of the high M-space children was completely successful on this task. The preponderance of errors made by the high M-space children were technique-oriented. Although an accurate technique had been demonstrated and practiced during the lesson, 11 of the 16 high M-space children used an approximate or careless form of iteration on the post-instruction task.

As in the post-instruction task in Lesson II, a simple perceptual strategy was used by at least some of the children in all of the developmental groups except for the high Piagetian--

high M-space group. While none of these eight children were completely successful, seven of them suffered only from a technique-based problem.

The remaining three cells of the developmental matrix were more difficult to characterize. Both low M-space cells contained children who used the most primitive strategies and children who used the most complete and accurate strategies. The logical reasoning abilities apparently had little effect on which measuring strategies children used.

Lesson Four. The post-instruction task in Lesson IV required children to construct a straight path equal in length to a polygonal path using unit rods of shorter length. The polygonal path was constructed with 7 cm. Cuisenaire rods and the children were given a collection of 5 cm. Cuisenaire rods. Two strategies were used most often on this task, one correct and one incorrect. The correct strategy required attention to both unit number and unit size, and consisted of laying out more short units to compensate for their smaller size. The most frequent incorrect strategy resulted from attending to only one dimension, unit number, and laying out "just as many" short units as there were long units.

While M-space did not seem to affect children's strategies on this task, logical reasoning ability did seem to have a definite effect. Thirteen of the 16 operational children attended to unit size as well as unit number in their solutions. In contrast, only two of the 16 preoperational children attended to both dimensions in their solutions. Both of these children were found to be in the transitional stage on the posttest Piagetian tasks.

Three of the 13 operational children who attended to unit size did not achieve a complete solution, but rather used a strategy which could be classified as transitional between recognizing only the number dimension and coordinating both dimensions of number and size. These three children recognized the difference in unit size but did not account for the sum of these differences. They suggested that a road with the same number of short units was the best solution given the materials, but that a completely accurate solution required the addition of "a little piece" equal in length to the difference of one pair of units (i.e., 2 cm).

Two strategies were used most frequently by the preoperational children. Seven of the eight low M-space children used a simple counting strategy and laid out just as many short rods as there were long ones. Four of the eight high M-space children constructed their road so that the endpoints of the two roads were aligned, i.e., they ignored the polygonal path of the first road. This solution required fewer short rods than long ones and was in some sense more primitive than the number strategy.

Effects of Instruction Problems on Post-Instruction Tasks

Each of the instruction lessons involved several instruction problems and a post-instruction task. The instruction problems focused on a single skill or concept and the post-instruction task represented the integration of these individual skills and concepts. Failure on the post-instruction task can be viewed as the result of a failure to master one or more of the prerequisites or as a failure to integrate them. Table 12 accounts for all failures on the four post-instruction tasks in one of these two ways. For purposes of table construction, "failure" was considered to be anything less than mastery (i.e., a score of 0 or 1 on a given problem or task).

The pattern of performance between columns is not markedly different. The high M-space group evidenced almost as much difficulty as the low M-space group in integrating the individual skills or concepts mastered during instruction.

A comparison of row performance suggests that a greater proportion of failures within the low Piagetian level group involved failure on prerequisite skills or concepts. As the analyses in Appendix A illustrate, some of these skills or concepts made direct demands on the logical reasoning abilities of conservation and transitivity.

Only in Lesson IV did the majority of failures on the post-instruction task involve mastery of all the prerequisites and a

Table 12

Failure on Post-Instruction Tasks in Terms of Prerequisite Skill Performance

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	A. 10 B. 9	A. 10 B. 8
	Low	A. 18 B. 4	A. 14 B. 9

A. Fail one or more instruction problems (prerequisites) and fail post-instruction task

B. Master all instruction problems (prerequisites) and fail post-instruction task

Table entries represent total number of occurrences failure to integrate or apply them. This was in spite of the fact that the instruction problems required a recognition of the principle upon which the post-instruction task was based. Many children were able to verbalize the inverse relationship between unit number and unit size and recognize this principle in facilitating contexts, but were not able to apply the principle to solve the post-instruction task.

Chapter VI

DISCUSSION

The primary purpose of this study was to examine the effect of several logical reasoning abilities and an information processing capacity on children's mathematics learning. The specific question of interest was: How do these developmental abilities affect children's ability to learn certain basic concepts and skills of linear measurement? A secondary purpose of the study was to explore the mediational role in this learning process of recognizing and resolving cognitive conflict. Is the ability to recognize and resolve conflict in a learning situation related to developmental level, and does this ability facilitate the learning of measurement concepts and skills? The first section of this chapter will provide some interpretation of the results with respect to each of these questions. The second section of the chapter will outline several limitations of the study, and the final two sections will suggest some implications of the study--for instruction and for future research.

Interpretation of Results

Logical reasoning ability. The logical task analysis carried out prior to instruction suggested that some of the measurement tasks on which children were instructed made significant demands on the ability to conserve length and reason transitively, and other measurement tasks did not. Differences between preoperational and operational children were predicted on the former set of logical-mathematical tasks but not on the latter set of technique tasks. The results supported these predictions.

Therefore, the ability to conserve and reason transitively did seem to affect children's learning of measurement. Furthermore, this effect was specific to certain concepts and skills. In other words, the Piagetian concepts of conservation and transitivity affect children's mathematics learning, but this effect depends on what is being learned. The distinction between technique-based tasks and logical-mathematical tasks seems to be a productive one in this regard. Children who possessed the logical reasoning abilities performed significantly better on the logical-mathematical tasks but not on the technique tasks. The implication of this result is that relationships between these logical reasoning abilities and learning mathematics concepts are specific rather than general. Certain Piagetian notions affect the ability to learn particular kinds of logically related mathematical concepts, but do not necessarily predict children's performance on all mathematical tasks. This conclusion is consistent with the synthesis of previous research presented in Chapter III, and helps to explain the conflicting results of earlier studies (compare, for example, Mpiangu & Gentile, 1975 with Steffe et al., Note 6).

The descriptive data, which focused on within-group performance on the measurement tasks, suggest a further refinement of this interpretation. The way in which the logical-reasoning abilities affected performance on the measurement tasks seemed to differ, even within the set of logical-mathematical tasks. Although the mean performance of the operational group exceeded the mean performance of the preoperational group on every logical-mathematical task, only the post-instruction task of Lesson IV was beyond the learning capabilities of the preoperational children. That is, at least some of the low Piagetian-level children were able to master most of the logical-mathematical tasks. Even though these children failed to conserve or reason transitively on the Piagetian tasks, they were able to carry out measurements which, from a logical perspective, required conservation or transitivity. Furthermore, the strategies which they used to measure logically required the application of these principles.

This finding illustrates the difficulty of developing a logical task analysis which matches the child's analysis of the task. The analysis of the post-instruction task in Lesson I, for example, included transitivity as a logical prerequisite. However some children who failed the transitivity task were able to complete this measurement task successfully. Obviously, the appropriate measurement strategy did not require the form of transitive reasoning which had been expected. Carpenter (in press-a) suggests that this kind of result further demonstrates the fact that children's logic is not congruent with adult logic. Children who do not possess conservation or transitivity are also incapable of reasoning that the absence of these abilities should have any consequences for their measuring behavior.

It may have been that the measuring skills which were taught during the lessons provided the children with techniques which allowed them to circumvent the logical concepts involved. In other words, children were able to learn and apply a measuring skill without thinking about the logical basis for its use.

Whatever the explanation for these findings, they do point out the deficiencies of the logical task analyses developed for this study. The results suggest that it may be difficult, in general, to carry out an adequate a priori task analysis. The observations of children's solution processes is suggested to be an essential part of any analysis which attempts to identify the component skills or concepts required to master a particular task.

One of the most striking results of the study was the poor performance of the preoperational children on the post-instruction task of Lesson IV. Only two of the 16 children scored above 0 on this task, and both of these children were found to be in transition toward concrete operations on the Piagetian post-tasks. Clearly, the logical reasoning abilities had a more pronounced effect on children's ability to master this task than they had on previous tasks.

An important difference between the Lesson IV post-instruction task and most of the other logical-mathematical tasks was that it required no new measuring technique for solution. Whereas the first three lessons included instruction on a measuring skill or technique which was needed to complete the tasks, Lesson IV focused on the logic of the inverse relationship between unit number and unit size. No new measuring technique was taught. It may have been that the preoperational children were not able to improve their understanding of this logical concept, and without a measuring skill or technique to compensate for this lack of understanding, they were unable to improve their performance on the task.

There is also another way to interpret these results. Carpenter (1975) notes that applying the inverse relationship between unit number and unit size to solve a measurement task requires the simultaneous coordination of two dimensions--number and size. In more general terms this requires an ability to decenter, to attend to several aspects of the problem at once rather than centering on only one dimension. This developmental ability, frequently emphasized by Piaget (1952, 1960; Piaget et al., 1960), is also required to complete the conservation and transitivity tasks. In this sense, the two Piagetian tasks and the post-instruction measurement task in Lesson IV are structurally similar. This similarity may have accounted for the near perfect predictability of performance on the measurement task from Piagetian operational level.

The results of this study further indicate that the decentering process required to complete these tasks is not acquired by specific instruction. The status studies of Carpenter (1975) and Carpenter and Lewis (1976) demonstrated that even though children recognize the inverse relationship between unit number and unit size, they are unable to coordinate these two dimensions in solving a conservation-type problem. The results reported here extend these findings by showing that, if children do not already possess this decentering ability (as measured by conservation and transitivity), they do not acquire it with specific instructional experiences. The descriptive data indicated that the majority of children who failed the post-instruction task had mastered the instruction problems. They had therefore learned to recognize that more of the smaller units were needed to measure a given length. But they were unable to use this information and coordinate these dimensions in constructing the required length.

Carpenter and Lewis (1976) hypothesize that "children do not develop the notion of the inverse relationship between unit size and number of units through experience measuring with different-sized units" (p. 57). The results of this study support this hypothesis. To the extent that measurement problems require the ability to decenter attention and coordinate several dimensions simultaneously they are unaffected by specific experiences in measuring. The ability to solve them seems rather to depend upon the development of basic cognitive abilities.

Information processing capacity. The results of this study indicate that information processing capacity, as measured by backward digit span, has no detectable effect on children's ability to learn measurement concepts and skills. The measure did not effectively discriminate between those children who mastered the measurement tasks and those who did not. Performance on the set of tasks where significant between-group differences were predicted showed that, while there was variation in these scores, the factor of information processing capacity accounted for 0% of the variation. Where significant between-group differences were found, they were in the "wrong" direction on a logically unrelated set of tasks. Low M-space children performed significantly better than high M-space children on the technique-based tasks. This result is difficult to explain and simply serves to reinforce the nonproductive nature of this measure in the present study.

Several problems accompanied the attempt to apply this relatively recent advance in cognitive psychology to the complex instructional setting employed here. These problems are basic ones and may explain the disappointing performance of the M-space construct. One of these problems is developing analysis procedures to determine the M-space demand of mathematical learning tasks. The studies reviewed in Chapter III indicate that M-space does predict performance on a variety of laboratory-type tasks--tasks for which it is possible to specify M-space demand. This kind of analysis requires an identification of the strategies which can be used to complete the task and a specification of the information processing capacity needed for the application of each strategy. The capacity required to apply a particular strategy must be determined from the child's point of view. It depends upon the sub-strategies which the child already has available, the familiarity of the task stimuli, the way in which the child approaches the task, etc. For artificial and novel tasks, this analysis can be carried out with a high degree of precision. However school mathematics tasks are much more complex and are confounded with previous experience. A given strategy may therefore make different demands ondifferent children. Because of these problems it was not possible to carry out this kind of refined analysis on the measurement tasks used in this study.

An alternate approach to analyzing the M-space demands of instructional tasks is suggested by Case (1975). This involves a logical task analysis and a specification of individual prerequisite skills or concepts. M-space is treated as an integration capacity and the M-demand of a particular task is determined by the number of individual skills which must be combined to complete the task. In this study, prerequisite skills were specified for each post-instruction measurement task. The instruction problems focused on individual prerequisite skills or concepts and the post-instruction tasks represented the integration of these prerequisites. Therefore it was assumed that the post-instruction tasks would place a higher demand on children's M-space. Consequently, high M-space children were expected to be more successful on these tasks than low M-space children. This was clearly not the case. The fact that many low M-space children mastered the post-instruction task suggests that the M-space demands of these tasks did not exceed their capacity. Low M-space may have been sufficient to complete both the instruction problems and the post-instruction tasks. The descriptive data support this hypothesis. Low M-space children had no more difficulty than high M-space children in putting together previously mastered prerequisite skills to complete the post-instruction tasks. Furthermore, the low coefficient of internal consistency for the post-instruction tasks (.22) suggests that this analysis procedure did not succeed in identifying a homogeneous set of tasks.

A second fundamental problem encountered in applying the M-space notion to an instructional context is one of identifying an appropriate measure of M-space. Backward digit span has been frequently used in the past (Case, 1974a, 1977; Lawson, 1976; Parkinson, 1975) and was used in this study. Intuitively, it is a valid measure since it requires two abilities which make up the M-space construct: short-term memory and an information operation or transforming element. The numbers in the task not only have to be held in mind, they have to be held in mind while operating on them in some way.

Based on this analysis, the backward digit span task seems to have a degree of face validity. However, it may not be a pure measure of M-space. The subjects in this study evidenced some variability on the 3-digit trials indicating the presence of some measurement error. The task may measure other attributes in addition to M-space. Although the amount of error is considered to be small, some subjects may have been misclassified. This introduces within-group error variance and reduces the likelihood of finding significant between-group differences.

An additional and fundamental question is whether a single general measure of M-space is appropriate. Recent work by Case and associates (Case, Kurland, & Daneman, Note 18) suggests that it may be very difficult to construct a general measure of M-space which will predict performance on a wide range of tasks. The data indicate that task variables, such as stimulus familiarity, may be more important than previously supposed in determining the M-space demand of a particular task. "Operational efficiency" is suggested to be as critical as M-space in predicting performance on a given task. Since operational efficiency depends on task variables and on the subject's available schemes or mental processes, the ability to apply a certain processing capacity will change from context to context. In this study, children who evidenced a high capacity on the backward digit span task may not have been able to apply this same high capacity to solve the measurement tasks, or children who were low on the backward digit span task may have had a high capacity within the measurement context. Clearly, the notion of operational efficiency will need to be accounted for in future attempts to develop measures of information processing capacity or M-space which have predictive validity.

The theory of M-space is intuitively promising. Many instructional tasks require the ability to combine several pieces of information in synthetic fashion rather than treating each piece independently. As a measure of this ability, the notion of M-space represents a fundamental cognitive capacity. Its usefulness in educational contexts, however, depends upon the possibility of developing an analysis procedure to specify the M-demand of complex learning tasks and a single (or multiple) measure of M-space which would predict children's performance on a given set of tasks. The results of this study suggest that additional basic research is needed before this will be achieved.

Significant results were obtained in this study with respect to logical reasoning ability but not with respect to information processing capacity. The difference in the productivity of these factors can be explained in part by the difference in their theoretical and empirical bases. A wealth of research exists within the Piagetian tradition which suggests important refinements in applying Piagetian constructs to educational settings. A similar research base has not yet been developed for the notion of M-space. Such a foundation may be needed before this construct can be usefully applied to instructional contexts.

Recognition and resolution of conflict. The cognitive conflict situations which were used in this study were relatively simple situations which represented modifications of those used by Inhelder et al. (1974). This form of conflict was selected for the study because of its instructional effectiveness in measurement situations (Inhelder et al., 1974; Carpenter & Hiebert, Note 2), and its usefulness in soliciting responses from subjects which indicate their understanding of the measurement concepts. Since the study employed only this one form of conflict, the results should not be generalized to other modes of cognitive conflict. Appropriate caution must therefore be exercised in interpreting the results.

Two questions were of interest in this study with respect to the role of recognizing and resolving cognitive conflict in learning measurement concepts. One was whether this ability was available only to the high developmental level children, and the second was whether this ability was an important one in learning measurement concepts and skills.

The first question was answered negatively. Low developmental level children were able to recognize and resolve conflict, and the degree to which they did so did not differ significantly from the high developmental children. This does not mean that the ability to deal with cognitive conflict is unrelated to developmental level. It only indicates that, given the way in which developmental level and conflict resolution were operationalized in this study, the capabilities of even the low developmental children were sufficient to recognize and resolve the specific conflict introduced during instruction.

The second question was answered in a more positive way. The ability to recognize and resolve conflict was found to account for a significant percentage (20.7%) of variation in children's measurement performance. Children who successfully dealt with the conflict situation were better able to improve their performance on the post-instruction tasks.

Some additional caution must be exercised in interpreting the results with respect to cognitive conflict. One advantage of clinical-type studies is the collection of observations gathered by the investigator which supplement the quantitative data. While these observations suffer from subjectivity and cannot be analyzed statistically, they do provide some insight into the phenomenon in question. In this case, these anecdotal observations suggest that the notion of cognitive conflict employed here is a difficult one to operationalize within the instructional setting and is a difficult one to measure. While many children were quite successful in explaining why different measurements were obtained, it was not clear whether cognitive conflict (in the Piagetian sense) had been generated or resolved. That is, it was difficult to assess whether some form of mental re-structuring had occurred.

The process of recognizing and resolving conflict is internally controlled. Children engage in this process when _they_ perceive some conflict between their observations and their internal conceptions. It is difficult to create external situations which automatically trigger this process, and it is difficult to know when it is occurring. Additional work is needed in refining this notion of conflict and in operationalizing it within instructional settings.

Limitations of the Study

Several limitations of the study have already been noted in the previous section, particularly with respect to the treatment of the M-space notion. Other limitations exist and will be described here.

Several characteristics of the sample selection procedure limit the interpretation of the results. First, children were excluded from the sample who were transitional with respect to their cumulative performance on length conservation and length transitivity. That is, children who passed length conservation but failed length transitivity, or vice versa, were excluded. This means that it was impossible to determine whether the effect of the logical reasoning abilities was due to either of these two abilities individually, or to an interaction between them. Further research must be conducted to isolate the effects of these individual abilities.

A second characteristic of the sample which must be considered was the bias introduced by selecting an equal number of subjects for each cell of the 2 X 2 developmental level matrix. This produced an orthogonal design with respect to logical reasoning ability and information processing capacity. To the extent that these two factors are related in the population, the sample used in this study was unrepresentative. If, for example, there is a high correlation between length conservation/length transitivity and backward digit span, then the high processing capacity/low reasoning ability and low processing capacity/high reasoning ability cells were over-represented. However, the lack of interaction effects between these two factors indicates that this unrepresentative nature of the sample need not interfere with the interpretation of the results with respect to the main effects.

A third characteristic of the sample also affected its representative nature. The three schools from which the sample was drawn did not contribute equally to each cell of the developmental level matrix. For example, one operational child and eleven preoperational children were drawn from one of the three schools. This unequal distribution reflects the fact that the majority of first-grade children in this school were at a low developmental level with respect to the logical reasoning abilities. A larger proportion of the children in the other two schools were at a high developmental level with respect to these abilities. This difference is possibly a function of the socio-economic status represented in the different school neighborhoods.

The assumption of this study was that the effects of socio-economic status and home experience were mediated through the specific factors of cognitive development which were employed. It was assumed that, given a particular instruction procedure and a particular level of prior knowledge, the factor which most directly affected children's mathematics learning was level of cognitive development. Consequently, external variables such as socio-economic status and school memebership were not included in the design.

An additional limitation of the design was the inclusion of transitional children with respect to information processing capacity. Children were classified as having a span of two or less digits, or a span of three or more digits. While the difference between a span of two and a span of three is quite substantial--theoretically it represents the difference of two developmental years (Pascual-Leone, 1970)--the scoring criteria arbitrarily sorted the transitional cases into one of the two categories. A selection procedure which would have identified and excluded transitional subjects would have yielded developmental groups which differed to a greater degree in information processing capacity.

Although such a procedure would have produced a design more sensitive to the effects of processing capacity, the data indicate that increased sensitivity would not have altered the findings. Since not even a trend was detected, it is doubtful that eliminating some random variance would have produced substantially different results. Furthermore, the high degree of stability of this factor demonstrated on the posttest suggests that only a few children were in a transitional stage. Therefore, it is believed that this limitation did not seriously affect the results.

Another limiting characteristic of the sample was the restriction to a single age group or developmental period. Since new developmental stages bring qualitative changes in intellectual abilities, they may also bring changes in the relationships between learning and development. Different relationships may exist with children of different ages and different developmental status. Consequently,the results of this study should be interpreted within the context of the particular age group used here.

This limitation is not overly severe, however, since the intent of the study was not to answer all of the questions of learning and development, but rather to investigate relationships between specific developmental abilities and specific mathematical content. Therefore the study was purposely restricted, not only to a particular age group, but to particular developmental abilities and particular mathematical concepts within that age group.

Consequently, the interpretations should not only be restricted to a particular age group, but also to the specific developmental abilities and mathematics content used here.

A final set of potentially limiting factors relates to the instruction procedure used. Several characteristics of the instruction procedure restrict its generalizability to other instructional settings. These were outlined in the last section of Chapter I and could be summarized by saying that this was not a study on instruction. It is not clear how other instruction procedures would have affected the results. It is tempting to say that conventional classroom instruction would not remove the specific constraints of development found in this study, but this is an empirical question.

One instructional issue requiring clarification which affects the internal validity of the study concerns the standardization of instruction. To what degree did all children receive the same instruction? This question is important since the interpretation given to the results depends upon attributing differences in performance to differences in development rather than to differences in instruction.

Completely standardized instruction was precluded by the individualized setting and the differential feedback given by the experimenter. Nevertheless, the instruction time and the opportunity to learn was maintained as constant as possible across subjects.

All subjects received the same tasks, all subjects were required to deal with conflict situations in each instruction problem, and all subjects were given the same opportunity to practice the newly-learned skills. Consequently, it was assumed that differences in performance did not result from differences in instruction.

Implications for Instruction

Mathematics instruction which is truly individualized must provide each student with appropriate mathematical tasks. The intent of this study was to investigate whether certain cognitive developmental abilities could be used to help determine "appropriateness." Presumably, mathematical concepts or skills which logically require certain reasoning abilities or processing capacity are beyond the learning capability of children who have not yet developed these abilities. This assumption represents one of the most fundamental implications of cognitive developmental psychology for the design of mathematics instruction. The present study is one of the few to carefully test the assumption.

Caution must be exercised, however, in drawing implications from this study for general mathematics instruction. Only one instructional procedure was used, and it differed in significant ways from conventional classroom instruction. It is not known how other kinds of instruction would have affected the results. In addition, the study focused on only several developmental abilities and on a limited set of mathematical concepts and skills.

Different relationships may exist between other developmental abilities and different mathematical content. To the extent that the following observations move beyond these limitations they should be regarded as hypotheses rather than conclusions.

1) The logical reasoning abilities identified by Piaget are required to learn certain kinds of mathematical concepts. Children in this study who did not yet conserve length or reason transitively were not able to use the inverse relationship between unit number and unit size in measurement contexts. Presumably, conservation and transitivity represent fundamental reasoning abilities which are needed to deal with certain mathematical ideas. Furthermore, the constraints imposed by the absence of these abilities are not removed by specific instruction. Although direct training was provided on similar tasks, the preoperational children did not improve their performance on the final task assessing their understanding of the inverse relationships between unit number and unit size.

2) The effects of these logical reasoning abilities on mathematics learning are specific rather than general. In fact, performance on Piagetian tasks appears to predict performance on only a narrow range of closely related mathematical tasks. Consequently, Piagetian tasks do not appear to be useful as general measures of learning readiness. In this study, conservation and transitivity were found to have no significant effect on learning many

mathematical tasks. For example, children who had not yet developed these abilities performed as well as high developmental children on the skill or technique-oriented tasks, and they were able to learn some of the logical-mathematical tasks.

3) A careful task analysis can be used effectively to identify the mathematical tasks which depend upon Piagetian logical reasoning abilities. In this study, tasks which did not make heavy demands on conservation and transitivity and which were skill-oriented were learned equally well by both developmental groups. Tasks which logically required these abilities but which involved the application of a learned measuring skill were mastered by some, but not all, of the low developmental children. Tasks which logically required these abilities and did not involve a skill component were not mastered by the low developmental children. Clearly, different types of tasks made different demands on the students' learning capabilities. An analysis of tasks along these lines is apparently a productive way to determine the appropriateness of the task for an individual learner.

4) The notion of M-space as measured by backward digit span has no immediate application to instructional settings. It is not clear what effect, if any, this capacity has on children's mathematics learning. Further research, as outlined in the next section, will need to refine this construct before it can be usefully applied to the selection of appropriate mathematical

content.

5) The problems associated with measuring the ability to recognize and resolve cognitive conflict make it of limited use as an independent variable in classroom settings. Although this ability did discriminate to a significant degree between those children who mastered the measurement tasks and those who did not, it is not proposed as a useful readiness measure. It was difficult to create situations which induced true cognitive conflict, and it was difficult to assess when such conflict was occurring. While this construct may be effectively employed in further research, it is of limited use as a readiness measure in the classroom.

Implications for Future Research

The results of the study indicate that relationships do exist between cognitive development and ability to learn mathematics, but that these relationships are specific to individual abilities and logically related or structurally similar mathematical tasks. Searching for a single, general measure of development which will predict children's performance on a wide variety of mathematical tasks may be a futile endeavor. However systematically documenting relationships between particular developmental abilities and learning logically related mathematical concepts will begin to build the "linking science" which Glaser (1976) had in mind. Such research will, in a cumulative way, establish a store of information about the effects of cognitive development on learning mathematics. This information can ultimately be used as a basis from which to select mathematical content which is appropriate for individual children.

Two lines of research will extend the findings of this study and contribute to linking cognitive developmental psychology to the design of mathematics instruction. First, relationships between developmental abilities and mathematics learning need to be established in other content domains. The work of Steffe et al. (Note 6) on early number concepts represents a step in this direction. Second, the effects of various instructional treatments on these relationships need to be investigated. Only one kind of treatment was used in this study. Further research should document the effect of other kinds of instruction on the relationships reported here.

A major contribution of the present study was the demonstration of the type of procedure which can be used to establish relationships between developmental abilities and mathematics learning. Future research should take into account the following observations.

The analysis of learning tasks is an important component of a successful procedure. In this study it was possible to carry out a detailed analysis of the demands made by each task on the logical reasoning abilities of conservation and transitivity.

This analysis successfully identified a set of tasks on which developmental group differences occurred. The analysis of tasks in terms of their information processing demand was conducted at a much lower level of specificity. This relatively superficial analysis did not successfully identify a corresponding set of tasks on which between-group differences occurred. A fine-grained task analysis, like the one conducted for the Piagetian variables, may be the key in uncovering the relationships which exist between particular developmental abilities and mastering mathematical tasks. Unfortunately, this level of analysis was not possible for the information processing variable in the present study due to the lack of knowledge about the information processing demand of complex mathematical tasks. Further research is needed within the psychological domain to identify the factors which affect the processing demand of various school mathematics tasks. Work on verbal learning by Kintsch and associates (Kintsch, Kosminsky, Streby, McKoon, & Keenan, 1975; Kintsch & van Dijk, 1978) and on general cognitive tasks by Sternberg (1977) suggest possible approaches to this problem.

A logical analysis is not the final step in specifying task demands. The results of this study showed that these analyses do not always match children's performance. Tasks which logically required certain reasoning abilities were successfully completed by some children who had not yet developed these abilities.

Observations of the strategies children use to solve the tasks are essential in understanding the demands which a given task places on individual children. Analysis of these strategies indicates what is required, from the children's viewpoint, to complete the task.

Another important component of a methodology designed to investigate relationships between development and mathematics learning is the selection of context-specific measures of cognitive development. In this study, length conservation and length transitivity were used successfully to investigate the learning of linear measurement. Both the developmental tasks and the learning tasks dealt with the attribute of length. Perhaps the importance of context-specificity acknowledged by Piaget (1972) for the formal operational level applies to other developmental periods as well. Cognitive tasks must be framed in the same context as the set of tasks on which performance is predicted.

The context of the task may be equally important for measuring information processing capacity. The nonsignificant results of this study with respect to this factor may have been due in part to the fact that information processing capacity was assessed using a number task while children's learning was assessed within a measurement context. The recent work of Case, Kurland, and Daneman (Note 18) points to the importance of using task stimuli which are similar to those in the tasks on which performance is predicted. Further research is needed to identify measures of information processing capacity which are specific to a given content domain.

Conclusion

A popular tradition or belief within the mathematics education community is that children's level of cognitive development affects their ability to learn mathematics. The results of the present study indicate that the tradition is justified, but only in part. Certain developmental abilities affect the learning of certain mathematical concepts, and this effect is evidenced in different ways. For some concepts the abilities appear to be essential, for others they are only facilitative, and for still others they are irrelevant. The complexity of these relationships underscores the futility of searching for general developmental measures which will predict performance on all mathematical tasks. Relationships are specific, and future research should be designed to systematically establish these relationships. Only with this bank of information can developmental differences between children be used effectively to select appropriate mathematical content.

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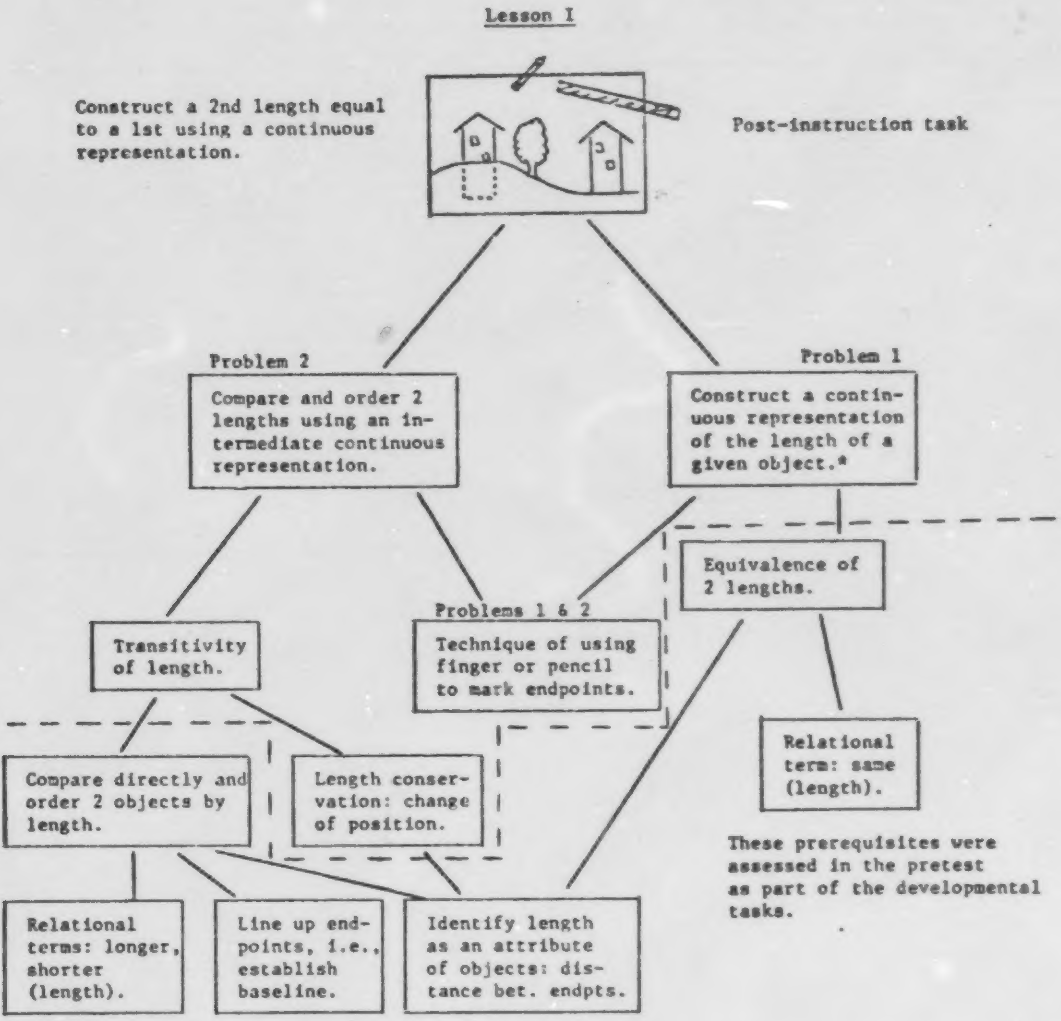
APPENDIX A

ANALYSIS OF POST-INSTRUCTION TASKS

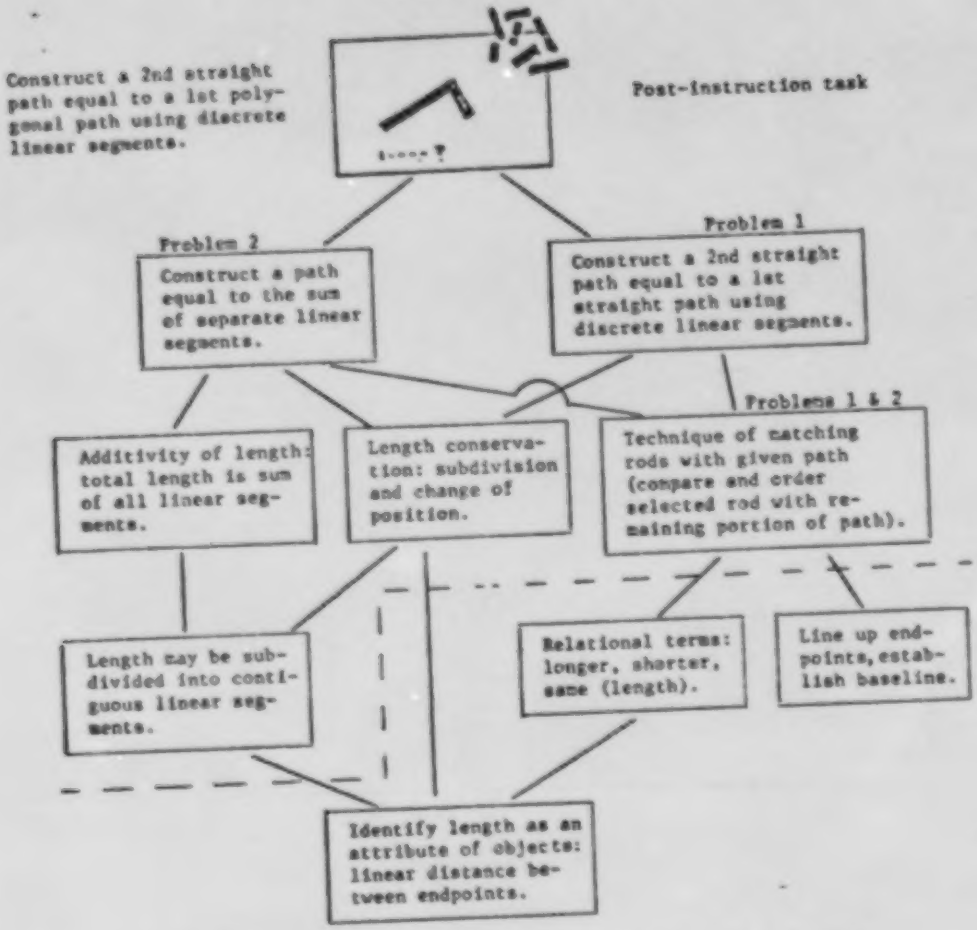
Analysis of Post-Instruction Tasks


Lesson I
Construct a 2nd length equal to a 1st using a continuous representation.
Post-instruction task
Problem 2
Compare and order 2 lengths using an intermediate continuous representation.
Problem 1
Construct a continuous representation of the length of a given object.*
Equivalence of 2 lengths.
Transitivity of length.
Problems 1 & 2
Technique of using finger or pencil to mark endpoints.
Relational term: same (length).
Compare directly and order 2 objects by length.
Length conservation: change of position.
Relational terms: longer, shorter (length).
Line up endpoints, i.e., establish baseline.
Identify length as an attribute of objects: distance bet. endpts.

These prerequisites were assessed in the pretest as part of the developmental tasks.

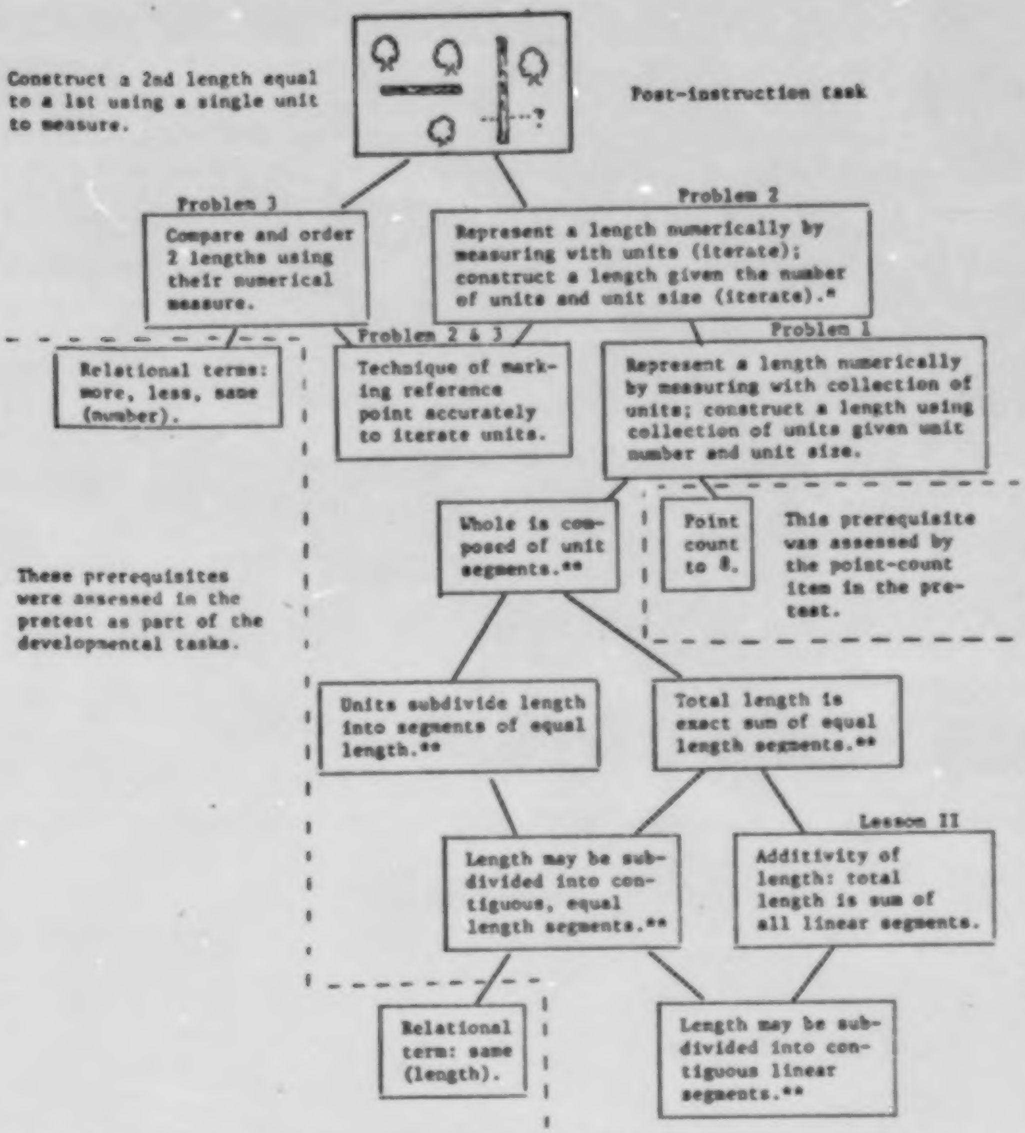
*Constructing the length representation of a given object and constructing an object of a given length are operationally synonymous.

Lesson II


Construct a 2nd straight path equal to a 1st polygonal path using discrete linear segments.
1---?
Post-instruction task
Problem 2
Construct a path equal to the sum of separate linear segments.
Problem 1
Construct a 2nd straight path equal to a 1st straight path using discrete linear segments.
Problems 1 & 2
Additivity of length: total length is sum of all linear segments.
Length conservation: subdivision and change of position.
Technique of matching rods with given path (compare and order selected rod with remaining portion of path).
Length may be subdivided into contiguous linear segments.
Relational terms: longer, shorter, same (length).
Line up endpoints, establish baseline.
Identify length as an attribute of objects: linear distance between endpoints.

These prerequisites were assessed in the pretest as part of the developmental tasks.

Lesson III


Construct a 2nd length equal to a 1st using a single unit to measure.

Post-instruction task

These prerequisites were assessed in the pretest as part of the developmental tasks.

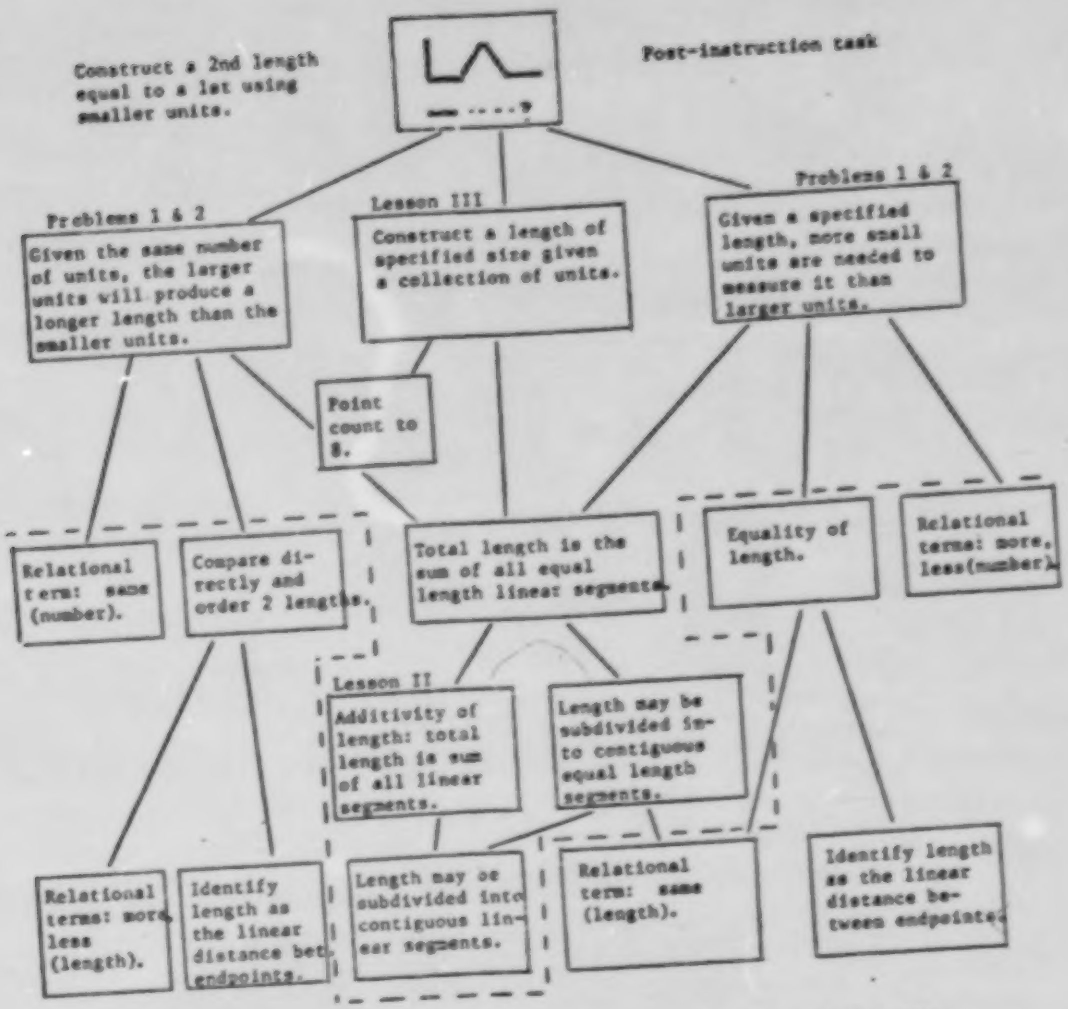
This prerequisite was assessed by the point-count item in the pretest.

?
Problem 3
Compare and order 2 lengths using their numerical measure.
Problem 2
Represent a length numerically by measuring with units (iterate); construct a length given the number of units and unit size (iterate).*
Relational terms: more, less, same (number).
Problem 2 & 3
Technique of marking reference point accurately to iterate units.
Problem 1
Represent a length numerically by measuring with collection of units; construct a length using collection of units given unit number and unit size.
Whole is composed of unit segments.**
Point count to 8.
Units subdivide length into segments of equal length.**
Total length is exact sum of equal length segments.**
Lesson II
Length may be subdivided into contiguous, equal length segments.**
Additivity of length: total length is sum of all linear segments.
Relational term: same (length).
Length may be subdivided into contiguous linear segments.**

*In this analysis these two processes are operationally synonymous, i.e., they share the same prerequisites.

** These were all included as a part of Problems 1 and 2.

Lesson IV


Construct a 2nd length equal to a 1st using smaller units.
Post-instruction task
Problems 1 & 2
Given the same number of units, the larger units will produce a longer length than the smaller units.
Lesson III
Construct a length of specified size given a collection of units.
Problems 1 & 2
Given a specified length, more small units are needed to measure it than larger units.
Point count to 8.
Relational term: same (number).
Compare directly and order 2 lengths.
Total length is the sum of all equal length linear segments.
Equality of length.
Relational terms: more, less(number).
Lesson II
Additivity of length: total length is sum of all linear segments.
Length may be subdivided into contiguous equal length segments.
Relational terms: more, less (length).
Identify length as the linear distance bet. endpoints.
Length may be subdivided into contiguous linear segments.
Relational term: same (length).
Identify length as the linear distance between endpoints.

These prerequisites were assessed in the pretest as part of the developmental tasks.

APPENDIX B

PRETEST TASKS AND SCORING CRITERIA

Pretest Tasks and Scoring Criteria

Point Counting (to 8)

E empties onto the table a cup containing 8 unifix cubes

COULD YOU COUNT THESE TO TELL ME HOW MANY THERE ARE?

COUNT THEM CAREFULLY

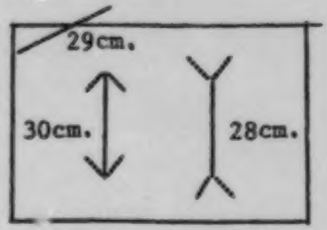
If S miscounts them E says

COULD YOU COUNT THEM AGAIN. THIS TIME COUNT THEM AS CAREFULLY AS YOU CAN AND TELL ME HOW MANY THERE ARE.

Scoring Criteria

Successful:	Correct response.
Unsuccessful:	Incorrect response.

Transitivity of Length



LET'S PLAY A LITTLE GAME WITH THESE STICKS.

E matches the green measuring stick with the longer of the two black ones.

ARE THESE TWO STICKS THE SAME LENGTH OR IS ONE OF THEM LONGER THAN THE OTHER? WHICH ONE?

SO THIS BLACK ONE IS LONGER THAN THE GREEN, AND THE GREEN IS LONGER THAN THIS BLACK ONE.

E removes the measuring stick and focuses attention to the table.

ARE THERE TWO STICKS THE SAME LENGTH OR IS ONE OF THEM LONGER? JUST LOOK AT THE STICKS, NOT THE THINGS ON THE END.

IS ONE OF THEM LONGER OR ARE THEY THE SAME?

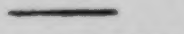
WHY DO YOU THINK SO?

Scoring Criteria

Successful:	Correct response and transitive reason.
Partially Successful:	Incorrect response and transitive reason.
Unsuccessful:	Visual comparison (correct or incorrect response).

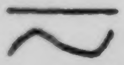
Conservation of Length

E lays out the two straight wires so that one pair of endpoints coincide.



LET'S PRETEND THAT THESE TWO WIRES ARE ROADS. IS THERE JUST AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS IT FARTHER ON ONE OF THE ROADS?

E bends longer road so the endpoints coincide.

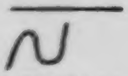


NOW IS THERE AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS ONE OF THE ROADS FARTHER?

(If the response is unclear or if the child does not seem to understand the question, rephrase it as follows.)

IF TWO ANTS ARE WALKING, ONE ON THIS ROAD AND ONE ON THIS ROAD, WOULD THEY BOTH WALK JUST AS FAR, OR WOULD ONE OF THEM WALK FARTHER?

E bends longer road so that the endpoints of shorter road extends beyond that of longer road.



NOW IS THERE AS FAR TO WALK ON THIS ROAD AS THIS ROAD, OR IS IT FARTHER ON ONE OF THE ROADS?

(Repeat clarification questions given above if necessary.)

Scoring Criteria

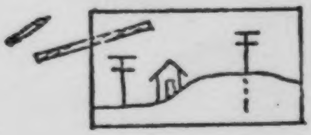
Successful:	Correct responses after both transformations.
Transitional	Incorrect response after first transformation, correct response after second transformation.

Unsuccessful:	Incorrect responses after both transformations, or correct response after first transformation and incorrect response after second transformation.

Pretest Measurement Tasks*

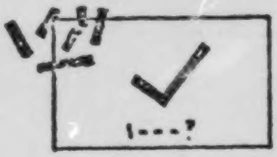
Task 1 HERE ARE TWO TELEPHONE POLES--THIS ONE MOVES UP AND DOWN. DO YOU THINK YOU CAN MOVE THIS POLE SO IT IS JUST AS BIG AS THE OTHER ONE?

YOU CAN USE THIS STRIP AND THE PENCIL TO HELP YOU MEASURE. USE THE STRIP TO HELP YOU MAKE SURE THAT BOTH POLES ARE JUST THE SAME SIZE.



Task 2 LET'S PRETEND THIS IS A ROAD FOR ANTS TO WALK ON. COULD YOU USE THESE RODS TO MAKE A STRAIGHT ROAD STARTING HERE WHICH HAS JUST AS FAR TO WALK AS THE CURVY ROAD.

MAKE SURE YOUR ROAD HAS JUST AS FAR TO WALK.



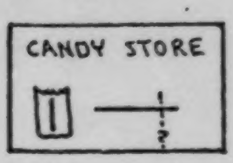
Task 3 HERE IS A PIECE OF LICORICE IN A BAG AND ANOTHER PIECE OF LICORICE IN THE STORE. LET'S PRETEND THAT YOU BOUGHT THIS PIECE AND YOUR FRIEND IS GOING TO BUY THIS PIECE. COULD YOU MEASURE THEM SO THAT YOUR FRIEND'S PIECE WILL BE JUST AS LONG

*The complete descriptions of these tasks are similar to those for the post-instruction tasks given in Appendix C. The scoring criteria for these tasks are identical to those for the post-instruction tasks given in Appendix E.

AS YOURS.

HERE IS A RED ROD TO HELP YOU MEASURE AND A SCISSORS WHICH YOU CAN USE TO CUT THE LICORICE SO THAT IT IS JUST AS LONG AS THE OTHER ONE.

USE THIS ROD TO MAKE SURE THAT YOU AND YOUR FRIEND WILL HAVE THE SAME AMOUNT TO EAT.



Task 4 WE ARE GOING TO BUILD SOME LITTLE ROADS FOR ANTS TO WALK ON.

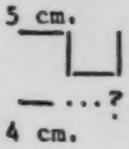
I WILL BUILD A ROAD WITH THESE YELLOW ONES.

NOW COULD YOU BUILD A STRAIGHT ROAD STARTING HERE WITH THESE PURPLE ONES SO THAT THERE IS JUST AS FAR TO WALK ON THE PURPLE ROAD AS ON THE YELLOW ROAD.

SUPPOSE TWO ANTS STARTED WALKING ON THESE ROADS, ONE HERE AND ONE HERE. WOULD THEY BOTH WALK JUST AS FAR?

HOW MANY RODS DOES YOUR ROAD HAVE? HOW MANY DOES MY ROAD HAVE?

SHOULD YOURS (MINE) HAVE MORE? (or--SHOULD THEY HAVE THE SAME NUMBER?)



Backward Digit Span

I WILL SAY SOME NUMBERS AND I WOULD LIKE YOU TO REPEAT THE SAME NUMBERS, ONLY YOU ARE TO SAY THEM BACKWARDS.

LISTEN CAREFULLY TO THE NUMBERS I SAY. THEN SAY THE SAME NUMBERS ONLY REMEMBER TO SAY THEM BACKWARDS.

LET'S PRACTICE A FEW.

E presents the following 3 series and provides correct responses for those which S answers incorrectly.

4, 2

8, 0

1, 6, 2

THAT'S GOOD. NOW WE'LL TRY SOME MORE. LISTEN CAREFULLY AND REPEAT THE NUMBERS YOU HEAR ONLY REMEMBER TO SAY THEM BACKWARDS.

Use the response sheet to read the digit series. For each series read one digit per second. Allow as much time as is needed between series.

Response Sheet

Backward Digit Span

Mark + for correct, 0 for incorrect.

7, 8 _____

0, 7 _____

4, 3 _____

5, 1 _____

6, 9 _____

8, 2 _____

5, 0 _____

1, 4 _____

9, 8 _____

5, 6 _____

7, 1, 3 _____

5, 8, 7 _____

8, 6, 2 _____

8, 1, 7 _____

0, 5, 3 _____

8, 4, 1 _____

2, 4, 3 _____

6, 2, 0 _____

1, 7, 6 _____

3, 8, 1 _____

Terminate the task after 3 consecutive errors.

Move to the next series after 6 consecutive correct reponses.

Scoring Criteria: Credit is given for a series after 6 consecutive correct responses or at least 7 correct out of 10 responses.

APPENDIX C

DESCRIPTION OF LESSONS

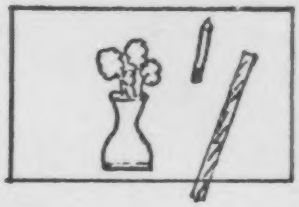
Description of Lessons

Lesson I

Instruction Problems

Problem 1

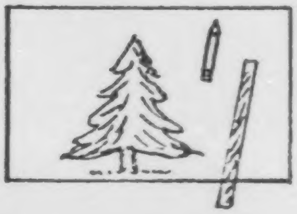
The experimenter (E) provided the subject (S) with a blank strip and a pencil and asked S to represent the height of the vase on the strip.



(All pictures used for these lessons were drawn on 8 1/2" x 11" tag-board)

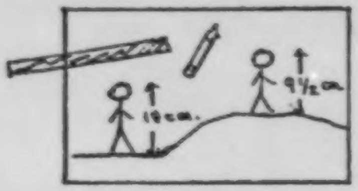
After S measured the vase, E measured it in a different way and obtained a different representation. If S had measured incorrectly, E measured correctly. If S had measured correctly, E measured incorrectly by dropping the endpoint of the strip below the bottom of the vase. S was asked to reconcile the different results. E then verbalized the important factors to be considered in constructing a representation such as this.

S was given a practice problem with instructions similar to those in the initial problem.



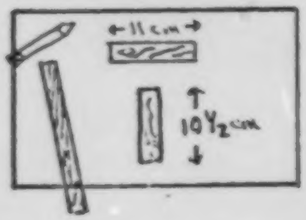
Problem 2

E provided S with a blank strip and a pencil and asked S to find out which of the two people was taller.



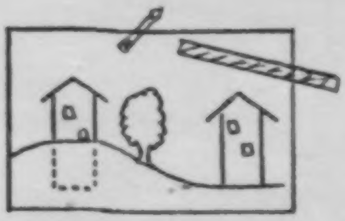
After S determined which person was taller E measured the people in a different way and obtained a different solution. If S had measured incorrectly, E measured correctly. If S had measured correctly, E measured incorrectly in a predetermined way. S was asked to reconcile the different results. E then verbalized the measurement principle involved in using an intermediate representation to compare and order the lengths of two objects.

S was given a practice problem with instructions similar to those in the initial problem.



Post-instruction Task

E provided S with a blank strip and a pencil and asked S to "build" the moveable building as big as the other one.

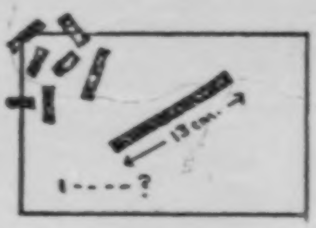


Lesson II

Instruction Problems

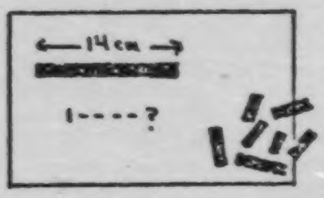
Problem 1

E provided S with a collection of various length Cuisenaire rods and asked S to build a "straight road" the same length as the given road, starting at the indicated point.



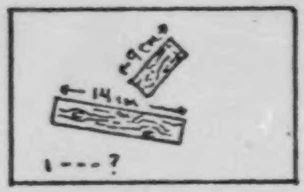
After S constructed a road, E constructed a second road using a different strategy and arrived at a different length. If S had produced an incorrect solution, E produced a correct solution by matching rods against the given road. If S had produced a correct solution, E produced an incorrect solution by vertically aligning the endpoint of the constructed road with the endpoint of the given road. S was asked to reconcile the different results. E then verbalized the measurement principle and indicated an appropriate comparison procedure which could be used to construct equal lengths.

S was given a practice problem with instructions similar to those in the initial problem.



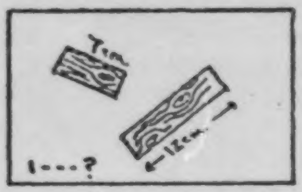
Problem 2

E provided S with a collection of various length square dowels (7 cm., 9 cm., 14 cm., and 17 cm.) and asked S to show how long the fence would be if the two boards were nailed together.



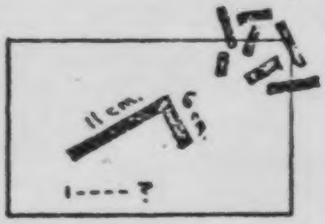
After S constructed a fence, E constructed a second fence using a different strategy and obtained a different result. If S had produced an incorrect solution, E produced a correct solution by matching dowel pieces against the two boards and "adding" them to form a straight fence. If S had produced a correct solution, E produced an incorrect solution in a predetermined way. S was asked to reconcile the different results. E then verbalized the additivity principle of measurement and indicated an appropriate measuring and matching strategy which could be used to add the measures of two lengths.

S was given a practice problem with instructions and materials similar to those in the initial problem.



Post-instruction Task

E provided S with a collection of various length Cuisenaire rods and asked S to build a "straight road" on which there was just as far to walk as on the crooked road, starting at the indicated point.

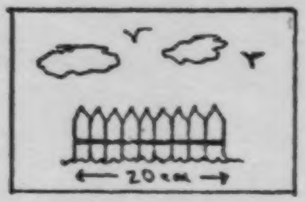


Lesson III

Instruction Problems

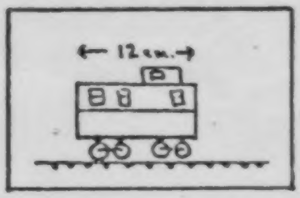
Problem 1

E provided S with a collection of 4 cm. Cuisenaire rods and asked S to measure the length of the fence.



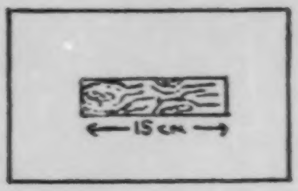
After S measured the fence, E measured it in a different way and obtained a different result. If S had measured incorrectly, E measured correctly. If S had measured correctly, E measured incorrectly by leaving a space between rods and getting a smaller measurement. S was asked to explain the reason for these different answers. E then verbalized the measurement principle involved in using a collection of units to measure a given length.

S was given a practice problem similar to the initial problem but using a collection of 3 cm. Cuisenaire rods.



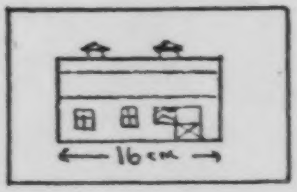
Problem 2

E provided S with one 3 cm. Cuisenaire rod and asked S to measure the length of the board.



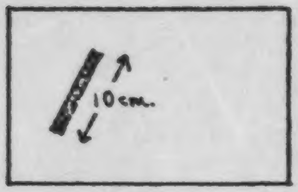
After S measured the board, E measured it in a different way and obtained a different result. If S had measured incorrectly, E measured correctly. If S had measured correctly, E measured incorrectly in a predetermined way. S was asked to reconcile the different results. E then verbalized the measurement principle involved in unit iteration and demonstrated a technique which can be used to iterate accurately.

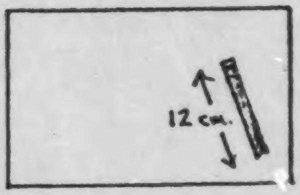
S was given a practice problem similar to the initial problem but using a 4 cm. Cuisenaire rod.



Problem 3

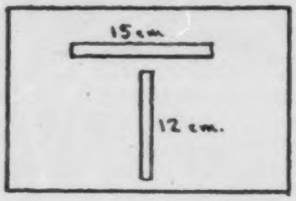
E provided S with one 2 cm. Cuisenaire rod and asked S to find out which of the two strips was longer. S was encouraged to use the rod to measure the strips.





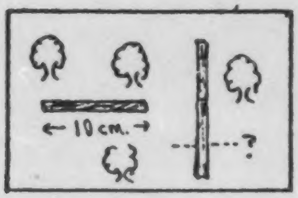
After S measured the strips, E measured them in a different way and obtained a different result. If S had measured incorrectly, E measured correctly. If S had measured correctly, E measured incorrectly by leaving space between iterations on the longer strip and obtaining a smaller measure. S was asked to reconcile the different results. E then restated the principle involved in unit iteration and verbalized the principle of comparing and ordering lengths by their unit measures.

S was given a practice problem similar to the initial problem but using a 3 cm. Cuisenaire rod.



Post-instruction Task

E provided S with a 2 cm. Cuisenaire rod and a scissors and asked S to make the second bike path in the park just as long as the first one. S was encouraged to use the rod to help measure.

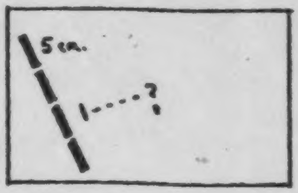


Lesson IV

Instruction Problems

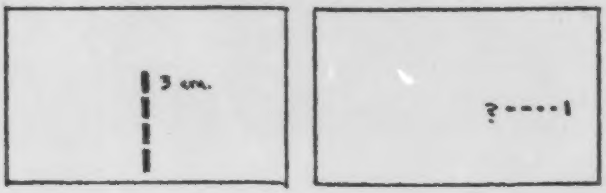
Problem 1

E constructed a "road" with four 5 cm. Cuisenaire rods. S was provided with a collection of 4 cm. Cuisenaire rods and asked to build a road just as long as E's road but starting at a different point and going in a different direction.



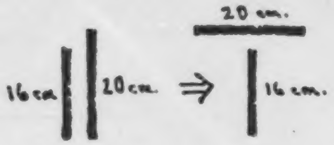
After S constructed the road, E moved the roads parallel to compare the lengths. S was asked to reconcile the fact that the same number of rods produced different lengths, and the fact that equal lengths required different numbers of rods. E then verbalized the measurement principle resulting from the inverse relationship between unit number and unit size.

S was given a practice problem similar to the initial problem but E's road was made with four 3 cm. Cuisenaire rods and S was given a collection of 4 cm. Cuisenaire rods.



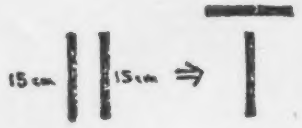
Problem 2

E placed two strips on the table, one 16 cm. and the other 20 cm. S was asked to compare their lengths. After S confirmed that one of them was longer, E moved them to form a "T".



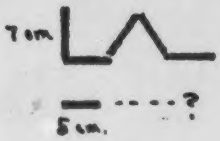
S was asked to measure the bottom strip using a collection of 4 cm. Cuisenaire rods and the top strip using a collection of 5 cm. Cusenaire rods. E then asked again about the relative length of the two strips. After S responded the strips were moved parallel and S was asked to explain the fact that the same number of rods were used to measure different lengths. E then restated the measurement principle resulting from the inverse relationship between unit number and unit size.

S was given a practice problem similar to the initial problem but using equal length strips (15 cm.) and measuring the bottom one with 3 cm. Cuisenaire rods and the top one with 5 cm. Cuisenaire rods.



Post-instruction Task

E constructed a "crooked road" with 7 cm. Cuisenaire rods. S was provided with a collection of 5 cm. Cuisenaire rods and asked to build a straight road on which there was just as far to walk.



APPENDIX D

LESSON PROTOCOLS

Lesson Protocols

Lesson 1

Problem 1 a. COULD YOU USE THIS STRIP AND THE PENCIL TO MEASURE HOW TALL THE VASE IS?

MARK WITH THE PENCIL ON THE STRIP TO SHOW JUST HOW TALL THE VASE IS.

b. SUPPOSE YOUR FRIEND MEASURED LIKE THIS (Either measure correctly by aligning bottom of strip with bottom of vase or measure incorrectly by dropping bottom of strip below bottom of vase).

WHY DID YOUR FREIND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY?

c. WHEN MEASURING HOW LONG SOMETHING IS WE NEED TO FIND THE HIGHEST AND LOWEST POINT AND MEASURE JUST FROM THE BEGINNING POINT TO THE ENDPOINT.

d. Practice (same as a. with respect to tree)

Problem 2 a. THESE TWO PEOPLE ARE HAVING AN ARGUMENT OVER WHICH ONE OF THEM IS TALLER. CAN YOU HELP THEM DECIDE WHO IS TALLER--YOU MAY USE THIS STRIP AND PENCIL TO HELP YOU MEASURE.

b. SUPPOSE YOUR FRIEND MEASURED LIKE THIS (Either measure correctly or measure incorrectly by measuring girl and then dropping bottom of strip below feet of boy to have boy appear taller).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY?

c. WE CAN USE THE SAME STRIP TO MEASURE BOTH PEOPLE AND FIND OUT WHO IS TALLER IF WE BEGIN AT THE SAME POINT ON THE STRIP FOR BOTH.

d. Practice A CARPENTER CUT THESE TWO BOARDS AND WOULD LIKE TO KNOW WHICH BOARD IS LONGER. YOU MAY USE THIS STRIP AND THIS PENCIL TO HELP YOU MEASURE.

Post-Instruction Task

HERE ARE TWO BUILDINGS--THIS ONE MOVES UP AND DOWN. DO YOU THINK YOU CAN MOVE THIS BUILDING SO IT IS JUST AS BIG AS THE OTHER ONE?

YOU CAN USE THIS STRIP AND THE PENCIL TO HELP YOU MEASURE. USE THE STRIP TO HELP YOU MAKE SURE THAT THERE IS JUST AS MUCH ROOM IN BOTH BUILDINGS.

Lesson 2

Problem 1 a. LET'S PRETEND THIS IS A ROAD FOR ANTS TO WALK ON. COULD YOU MAKE A STRAIGHT ROAD STARTING HERE WHICH IS JUST AS LONG AS THE OTHER ROAD?

MAKE YOUR ROAD SO THAT BOTH ANTS WILL HAVE JUST AS FAR TO WALK.

IS YOUR ROAD JUST AS FAR?

b. SUPPOSE YOUR FRIEND MADE THE ROAD LIKE THIS (Either measure correctly by matching rods along strip and laying out road or measure incorrectly by using a perceptual strategy and aligning endpoint of second road with endpoint of first road).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY?

c. WHEN YOU NEED TO BUILD A ROAD WHICH IS JUST AS LONG, YOU CAN MAKE SURE BY MATCHING YOUR ROAD WITH THE OTHER ONE.

d. Practice (same as a.) MAKE SURE THAT BOTH ROADS HAVE JUST AS FAR TO WALK. DO YOU THINK YOUR ROAD IS JUST AS FAR?

Problem 2 a. THESE ARE TWO REDWOOD BOARDS. A CARPENTER IS GOING TO BUILD A FENCE WITH THEM BY NAILING THEM TOGETHER.

COULD YOU HELP THE CARPENTER BUILD THE FENCE?
USE THESE TO SHOW HOW LONG THE FENCE WOULD BE IF
THESE TWO BOARDS WERE NAILED TOGETHER.

b. (If S measured incorrectly)
SUPPOSE YOUR FRIEND MEASURED LIKE THIS
(Match rods correctly and lay out fence).
WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?
WHO DO YOU THINK IS RIGHT? WHY?
(If S measured correctly)
SUPPOSE YOUR FRIEND MEASURED LIKE THIS
(Lay long rod and show endpoint is vertically
aligned with one of the boards).
WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?
WHO DO YOU THINK IS RIGHT? WHY?

c. WHEN WE WANT TO FIND OUT HOW LONG TWO THINGS ARE
TOGETHER WE CAN MEASURE EACH ONE AND THEN ADD
THEM TOGETHER.

d. (Same as a. with respect to building a bench)

__Post-Instruction Task__

LET'S PRETEND THIS IS A ROAD FOR ANTS TO WALK ON.
COULD YOU MAKE A STRAIGHT ROAD STARTING HERE WHICH
HAS JUST AS FAR TO WALK AS THE CURVY ROAD?
__MAKE SURE__ YOUR ROAD HAS JUST AS FAR TO WALK.

Lesson 3

Problem 1 a. COULD YOU MEASURE HOW LONG THIS FENCE IS USING THESE?

4 cm. rods HOW LONG IS THE FENCE? HOW MANY OF THESE?

b. SUPPOSE YOUR FRIEND MEASURED THE FENCE LIKE THIS (Either measure correctly, or incorrectly by leaving space between each unit resulting in 4 units rather than 5).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY? (If S doesn't recognize conflict measure again to get 3 units and repeat questions)

c. WHEN MEASURING WITH RODS LIKE THIS WHICH ARE ALL JUST THE SAME WE NEED TO KNOW HOW MANY OF THEM IT TAKES TO GO FROM BEGINNING TO END SO THAT THE WHOLE FENCE IS COVERED--WITH NO SPACE BETWEEN. ONE ROD MUST BEGIN WHERE THE LAST ONE ENDED.

3 cm. rods d. Practice (same as a. with respect to train car)

Problem 2 a. COULD YOU FIND OUT HOW LONG THIS BOARD IS? USE THIS ROD TO MEASURE IT.

3 cm. rod b. 1. (If S measured correctly)

SUPPOSE YOUR FRIEND MEASURED IT LIKE THIS (Measure incorrectly by visually estimating transition points, leaving space between, and getting less units as a result).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY?

2. (If S measured incorrectly) SUPPOSE YOUR FRIEND MEASURED IT LIKE THIS (Measure correctly using finger to mark reference points).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY?

c. WHEN WE USE ONE ROD TO MEASURE WE MUST BE CAREFUL TO START THE NEXT ONE RIGHT WHERE THE LAST ONE STOPPED SO THAT WE MEASURE THE WHOLE BOARD.

ONE WAY WE CAN DO THIS IS TO USE OUR FINGER OR A POINTER TO REMEMBER WHERE THE LAST ONE STOPPED—LIKE THIS (Measure several units along board).

PUT YOUR FINGER BESIDE THE ROD, NOT IN FRONT OF IT.

d. Practice (same as a. with respect to barn) MEASURE
4 cm. rod THIS ONE VERY CAREFULLY.

Problem 3 a. LET'S FIND OUT WHICH OF THESE TWO STRIPS IS LONGER.
2 cm. rod COULD YOU MEASURE THEM TO FIND OUT WHICH IS LONGER?
HERE IS A ROD TO HELP YOU MEASURE.

b. 1. (If S measured correctly)

SUPPOSE YOUR FRIEND MEASURED IT LIKE THIS (Measure 10 cm. strip (on left) correctly by visually determining marking points and 12 cm. strip incorrectly by visually estimating and leaving space between rods to get 4 units).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT? WHY?

2. (If S measured incorrectly) SUPPOSE YOUR FRIEND MEASURED IT LIKE THIS (Measure correctly using finger to mark reference point).

WHY DID YOUR FRIEND GET A DIFFERENT ANSWER?

WHO DO YOU THINK IS RIGHT?

WHY?

c. WHEN WE MEASURE WITH ONE ROD WE MUST BE CAREFUL TO START THE NEXT ONE RIGHT WHERE THE LAST ONE STOPPED. IF WE MEASURE CAREFULLY USING THE SAME ROD WE CAN FIND OUT WHICH THING IS LONGER BY COUNTING HOW MANY RODS.

3 cm. rod

Post-Instruction Task

2 cm. rod

d. Practice (same instructions as a. with respect to "T")

HERE IS A BIKE PATH RUNNING THROUGH THE PARK. THIS IS ANOTHER PATH THAT THEY ARE JUST MAKING. CAN YOU HELP THEM BUILD THE PATH SO THERE WOULD BE JUST AS FAR TO RIDE ON THIS PATH AS ON THAT ONE.

HERE IS A SCISSORS WHICH YOU CAN USE TO CUT THE PATH SO THAT IT IS JUST AS LONG AS THE OTHER ONE.

YOU CAN USE THIS RED ROD TO HELP YOU MEASURE.

Lesson 4

Problem 1 a. I'M GOING TO BUILD A ROAD WITH THESE YELLOW ONES WHICH LOOKS LIKE THIS (4 rods).

COULD YOU BUILD A STRAIGHT ROAD STARTING HERE WITH THE PURPLE ONES SO THAT YOUR ROAD IS JUST AS LONG AS THIS ONE? BE SURE THAT THERE WOULD BE JUST AS FAR TO WALK ON YOUR ROAD AS THERE IS ON THIS ONE.

IS YOUR ROAD JUST AS LONG NOW?

b. HOW MANY RODS ARE IN YOUR ROAD? HOW MANY IN THIS ONE?

DO YOU THINK BOTH ROADS ARE JUST AS LONG?

LET'S CHECK THEM.

(Move yellow road parallel to purple road)

1. (_If_ both roads were 4 rods) WHY DO YOU THINK THE ROADS ARE DIFFERENT IF THEY ARE BOTH 4 RODS LONG?

2. (_If_ roads were same length) WHY DOES ONE HAVE 4 RODS AND THE OTHER 5 RODS?

3. (_If_ roads were different lengths) COULD YOU MAKE THE PURPLE ROAD JUST AS LONG?

WHY IS ONE ROAD 5 RODS LONG AND THE OTHER ROAD 4 RODS LONG?

c. SO IF ONE OF THE RODS IS SHORTER WE NEED TO USE MORE OF THEM TO MAKE THE SAME LENGTH.

d. _Practice_ (same as a. with 4 light green rods and purple rods for S)

MAKE SURE YOUR ROAD IS JUST AS LONG.

Problem 2 a. WHICH OF THESE STRIPS IS LONGER OR ARE THEY THE SAME?

(Place strips from ||into⊤)

COULD YOU MEASURE THE BOTTOM STRIP WITH THE PURPLE ONES?

SO THE BOTTOM ONE IS 4 PURPLES.

COULD YOU MEASURE THE TOP STRIP WITH THE YELLOW ONES?

SO THE TOP STRIP IS 4 YELLOW ONES.

IS ONE OF THE STRIPS LONGER OR ARE THEY THE SAME?

b. 1. (If S says they are different) WHY? BUT WHY DID THEY MEASURE THE SAME NUMBER?

2. (If S says they are the same) WHY? BUT ONE OF THEM WAS LONGER BEFORE--WHY DO YOU THINK IT'S THE SAME NOW?

3. (Move top strip and rods parallel) WHICH STRIP IS LONGER?

WHY DO YOU THINK THEY MEASURE THE SAME NUMBER OF RODS?

c. IF WE WERE GOING TO MAKE THEM THE SAME LENGTH WE WOULD NEED MORE PURPLE RODS. SO IF YOU ARE USING SHORTER RODS TO MEASURE SOMETHING YOU WILL NEED MORE OF THEM TO MEASURE THE SAME LENGTH.

d. Practice WHICH OF THESE STRIPS IS LONGER OR ARE THEY THE SAME? (Move strips from||to⊤)

COULD YOU MEASURE THE BOTTOM STRIP WITH THE LIGHT GREEN RODS?

SO THE BOTTOM ONE IS 5 LIGHT GREEN RODS.

COULD YOU MEASURE THE TOP STRIP WITH THE YELLOW RODS?

SO THE TOP STRIP IS 3 YELLOW RODS.

IS ONE OF THE STRIPS LONGER OR ARE THEY THE SAME?

WHY?

Post-Instruction Task

WE ARE GOING TO BUILD SOME LITTLE ROADS FOR ANTS TO WALK ON. I WILL BUILD A ROAD WITH THESE BLACK ONES. NOW COULD YOU BUILD A STRAIGHT ROAD STARTING HERE WITH THESE YELLOW RODS SO THAT THERE IS JUST AS FAR TO WALK ON THE YELLOW ROAD AS THE BLACK ROAD?

SUPPOSE TWO ANTS STARTED WALKING ON THESE ROADS, ONE HERE AND ONE HERE. WOULD THEY BOTH WALK JUST AS FAR? HOW MANY DOES YOUR ROAD HAVE? HOW MANY DOES MY ROAD HAVE? SHOULD YOURS (MINE) HAVE MORE? (or--SHOULD THEY HAVE THE SAME NUMBER?)

APPENDIX E

CODING SCHEMES AND SCORING CRITERIA

Coding Schemes and Scoring Criteria

Coding Scheme--Lesson I--Problem 1

1. Did not use strip--did not mark beginning and/or endpoints.
2. Marked off a segment corresponding to only a part of the length.
3. Marked off entire length but did not attend to endpoints:
    a) did not match highest and/or lowest point.
    b) placed strip off to the side and estimated points.
4. Marked off length correctly.

Scoring Criteria

Unsuccessful (0):	Did not represent length on the strip, evidenced little understanding of this concept (strategies 1 and 2).
Partially Successful (1):	Represented length on the strip but result only approximate (strategy 3).
Successful (2):	Measured correctly--attended to both endpoints (strategy 4).

Coding Scheme--Lesson I--Problem 2

1. Did not use strip--perceptual or other solution.
2. Used strip but only perceptually or otherwise (e.g., used horizontally to align endpoints).
3. Used strip to measure one length or the other (correctly or incorrectly) but did not compare them.
4. Used strip to compare lengths but incorrect measurement resulted in erroneous conclusion:
    a. line up end of strip with end of page.
    b. did not attend to endpoints.
    c. strip was incorrectly placed for comparison of second length.
5. Correct measurement and comparison.

Scoring Criteria

Unsuccessful (0);	Did not use the strip to compare the 2 lengths (strategies 1-3).
Partially Successful (1):	Compared lengths with the strip but incorrect procedure led to erroneous conclusion (strategy 4).
Successful (2):	Correct measurement and comparison (strategy 5).

Coding Scheme--Lesson I--Post-Instruction Task

1. Did not use strip--perceptual or other solution.
2. Used strip but only perceptually or otherwise (e.g., laid strip horizontally).
3. Used strip to measure one length but not the other.
4. Used strip to measure both lengths but did not measure the lengths themselves.
5. Used strip to measure both lengths but adjusted by perceptual judgment.
6. Used strip to measure both lengths but did not attend to both pairs of endpoints.
7. Measured correctly.

Scoring Criteria

Unsuccessful (0):	Used a perceptual strategy or used strip but did not evidence an understanding of measurement principles (strategies 1-4).
Partially Successful (1):	Measured both lengths, but achieved only an approximate result (strategies 5 and 6).
Successful (2):	Measured correctly--attended to both pairs of endpoints (strategy 7).

Coding Scheme--Lesson II--Problem 1

1. Perceptual solution--"just looks right."
2. Perceptual solution--aligned endpoint of second road vertically with endpoint of first.
3. Perceptual solution--then compared (and corrected second road) by "measuring" both with intermediate object (finger span, iteration with rod, etc.):
    a. used gross visual estimate.
    b. used marking or careful visual estimate procedure.
4. Matched rods to the given road but only in approximate fashion (e.g., matched one rod at a time and visually estimated reference points on given road; matched rods to only part of the road).
5. Matched rods, laid out road, then changed it according to perceptual judgment.
6. Matched rods in trial and error fashion--laid out straight road, then matched and corrected road, then matched road again, etc.
7. Matched rods systematically and correctly.

Scoring Criteria

Unsuccessful (0): Evidenced no understanding of measurement principles, i.e., used strategies based on perceptual judgment (strategies 1-3a).

Partially Successful (1):	Evidenced some understanding of the measurement principles by matching rods to the given road or by using some other measurement technique but reached only an approximate solution (strategies 3b-5).
Successful (2):	Evidenced an understanding of the measurement principles and reached an exact solution (strategies 6 and 7).

Coding Scheme--Lesson II--Problem 2

1. Perceptual solution--"just looks right".
2. Perceptual solution--aligned endpoint of constructed length vertically with endpoint of one of the boards.
3. Perceptual solution--then compared by "measuring" with an intermediate object (finger span, etc.) but used only visual estimates:
   a. measured only one board; or both boards treated individually.
   b. measured both boards and used their sum for comparison.
4. Matched rods with one board or the other, or both individually, but did not combine results to construct length.
5. Matched rods with both boards, combined results to construct length, then adjusted length by perceptual judgment.
6. Matched rods with both boards in trial and error fashion--laid out length, then matched rods with boards and corrected length, etc.
7. Matched rods with both boards and combined results to construct length.

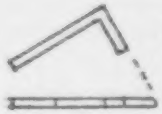
Scoring Criteria

Unsuccessful (0):	Used only perceptual solution and/or evidenced no understanding of the additivity principle (strategies 1, 2, 3a, and 4).
Partially Successful (1):	Evidence some understanding of the additivity principle but reached

	only an approximate solution (strategies 3b and 5).
Successful (2):	Evidenced an understanding of the additivity principle and reached an accurate solution (strategies 6 and 7).

Coding Scheme--Lesson II--Post-Instruction Task

1. Perceptual solution--"just looks right."
2. Perceptual solution--aligned endpoint of second road vertically with endpoint of first.
3. Perceptual solution--aligned endpoint of second road angularly with endpoint of first.



4. Perceptual solution--then compared by "measuring" both with intermediate object (finger span, iteration with rod, etc.):
    a. used gross visual estimate.
    b. used marking or careful visual estimate procedure.
5. Matched rods to the given road but only in approximate fashion (e.g., matched one rod at a time and visually estimated reference points on given road; matched rods to only part of the road).
6. Matched rods, laid out road, then corrected it according to perceptual judgment.
7. Matched rods in trial and error fashion--laid out straight road, then matched and corrected road, then matched road again, etc.
8. Matched rods by laying them beside given road.
9. Matched rods correctly.

Scoring Criteria

Unsuccessful (0):	Evidenced no understanding of measurement principles, i.e., used strategies

	based on perceptual judgement (strategies 1-4a).
Partially Successful (1):	Evidenced some understanding of the measurement principles by matching rods to the given road or by using some other measurement technique but reached only an approximate solution (strategies 4b-6).
Successful (2):	Evidenced an understanding of the measurement principles (strategies 7-9).

Coding Scheme--Lesson III--Problem 1

1. Laid rods along only part of the object--did not cover the whole length (e.g., did not go from endpoint to endpoint; left spaces between rods).
2. Laid rods along entire length but failed to count them appropriately (e.g., counted only some of them; counted them twice). (This does not include an accidental miscount.)
3. Used a basically correct measuring and counting procedure but did not carefully attend to the endpoints.
4. Used correct procedure and achieved an accurate solution.

Scoring Criteria

Unsuccessful (0):	Evidenced little understanding of unit measurement and assigning a numerical value to a specified length (strategies 1 and 2).
Partially Successful (1):	Evidenced some understanding of unit measurement but did not achieve an accurate solution (strategy 3).
Successful (2):	Evidenced an understanding of unit measurement and achieved an accurate solution (strategy 4).

Coding Scheme--Lesson III--Problem 2

1. Measured length without using rod for iteration (e.g., finger span, steps with fingers or with rod as pointer, etc.).
2. Measured length by sliding rod along while counting or "stepping" rod along length without attending to reference points.
3. Measured length with rod by using entire finger as the reference point:
   a. counted finger as well as rod to give measure.
   b. counted only rod movements to give measure.
4. Measured length with rod by visually and carefully estimating reference points.
5. Measured length with rod using appropriate technique but miscounted in some way.
6. Measured length with rod correctly by accurately marking reference points with finger.

Scoring Criteria

Unsuccessful (0):	Evidenced little or no understanding of unit iteration as a measurement process (strategies 1, 2, and 3a).
Partially Successful (1):	Evidenced some understanding of unit iteration but used inaccurate technique or miscounted (strategies 3b, and 5).

Successful (2):	Evidenced an understanding of unit iteration and used an accurate technique (strategies 4 and 6).

Coding Scheme--Lesson III--Problem 3

1. Perceptual solution--one looks longer (did not measure).
2. Measured only one length with rod, i.e., did not compare length by measuring both.
3. Measured lengths using procedures other than unit iteration (e.g., finger span, steps with fingers).
4. Measured both lengths using approximate unit iteration and based response on this measurement:
    - a. slid rod along length or "stepped" rod along length while counting--did not use reference points.
    - b. used entire finger as the reference point.
    - c. miscounted in some way.
5. Measured both lengths but gave response based on perceptual judgment rather than a comparison of the measures.
6. Measured both lengths correctly (attended to reference points by careful visual estimates or marking them with fingers) and gave appropriate response.

Scoring Criteria

Unsuccessful (0):	Used only perceptual or gross unit iteration procedures to measure and/or did not use the measures to compare two lengths (strategies 1-3, 4a, and 5).
Partially Successful (1):	Used an approximate form of unit iteration and compared length based

	on these measures (strategies 4b and 4c).
Successful (2):	Measured accurately by iterating a unit and used results to correctly compare lengths (strategy 6).

Coding Scheme--Lesson III--Post-Instruction Task

1. Perceptual solution--"looks just as long" (did not measure).
2. Measured only one length with rod, i.e., did not compare strips by measuring both.
3. Measured second length in wrong direction and was left with complement.
4. Measured lengths using procedures other than unit iteration (e.g., finger span, steps with fingers).
5. Measured both lengths using approximate form of unit iteration (systematic or trial and error):
    a. slid rod along lengths or "stepped" rod along length while counting--did not use reference points.
    b. used entire finger as the reference point.
    c. miscounted in some way.
6. Measured both lengths using an accurate technique and unit iteration--careful visual estimates or finger marking (systematic or trial and error).

Scoring Criteria

Unsuccessful (0):	Used only perceptual or gross unit iteration procedures to measure and/or did not use the measures to compare two lengths.
Partially successful (1):	Used an approximate form of unit iteration and compared lengths based

	on these measures (strategies 5b and 5c).
Successful (2):	Measured accurately by iterating a unit and used results to correctly compare lengths (strategy 6).

Coding Scheme--Lesson IV--Problem 1

1. Perceptual solution--"looks just as long."
2. Counting solution--laid out *just as many* rods.
3. Attended to unit size, i.e., recognized the difference in unit size and indicated that this was a relevant factor in the solution but did not use the information appropriately (e.g., laid out *more* long rods than short).
4. Matched rods alongside given road:
    a. changed final road using perceptual judgment.
    b. used matching to achieve an accurate solution.
5. Attended to unit size and laid out less long rods than short ones (or more short rods than long ones).

Scoring Criteria

Unsuccessful (0):	Built road without accounting for unit size in the construction process (strategies 1, 2, and 4a).
Partially Successful (1):	Accounted for unit size in construction of road but did not use the inverse relationship appropriately (strategy 3).
Successful (2):	Accounted for unit size in appropriate way to achieve approximate or accurate solution, or achieved accurate solution by using matching strategy (strategies 4b and 5).

Coding Scheme--Lesson IV--Problem 2

1. Gave incorrect response with following explanation:
   - a. no reason given.
   - b. perceptual explanation.
   - c. number of units.
2. Gave correct response with following explanation:
   - a. no reason given.
   - b. perceptual explanation.
   - c. they were the same (different) size before.
   - d. different-size rods measure different number of units.

Scoring Criteria

Unsuccessful (0):	Gave incorrect response (strategy 1).
Partially Successful (1):	Gave correct response but did not provide logical explanation (strategies 2a and 2b).
Successful (2):	Gave correct response and provided logical explanation (strategies 2c and 2d).

Coding Scheme--Lesson IV--Post-Instruction Task

1. Did not make straight road (thought an equal-length road must be crooked).
2. Perceptual solution--"just looks right."
3. Perceptual solution--aligned endpoint of second road vertically with endpoint of first road.
4. Perceptual solution--then compared (and corrected second road) by "measuring" both with an intermediate object (e.g., finger span).
5. Counting solution--used as many short rods as there were long ones.
6. Attended to both unit size and unit number in constructing a solution:
    a. adjusted solution according to perceptual judgment.
    b. attended to the difference in unit size but did not account for the sum of these differences.
    c. considered size ratio and used approximately correct number of short rods (from 6 to 9).

Scoring Criteria

Unsuccessful (0):	Evidenced no understanding of the inverse relationship between unit size and unit number, i.e., used strategies based on perceptual judgment or on unit number only (strategies 1-5).

Partially Successful (1): Recognized the inverse relationship but did not achieve an approximately accurate solution (strategies 6a and 6b).

Successful (2): Recognized the inverse relationship and achieved an approximately accurate solution (strategy 6c).

Scoring Criteria for Recognizing and Resolving Conflict

Unsuccessful (0):	Did not recognize or notice the difference in results or did not see the inconsistency in arriving at two different measures for the same length (e.g., "both are right").
Partially Successful (1):	Indicated that they noticed a difference in the results and that one of the results must be incorrect (either by spontaneous verbal statement, verbal response to question, or demonstration); but could not explain or otherwise reconcile the difference, or explained it on the basis of a non-measurement rationale.
Successful (2):	Recognized and explained the difference in results either verbally or via demonstration by appealing to the relevant principle(s) or measurement, either directly or indirectly.

APPENDIX F

CONTINGENCY TABLES--

DEVELOPMENTAL GROUPS X MEASUREMENT TASK PERFORMANCE

Developmental Groups X Measurement Task Performance

Logical Reasoning Ability

Task	Table				Mean	S.D.
		Task Score				
		0	1	2		
Lesson I, Problem 1	High	0	5	11	1.6875	.4787
	Low	0	5	11	1.6875	.4787
		0	1	2		
Lesson I, Problem 2*	High	2	1	13	1.6875	.7042
	Low	1	7	8	1.4375	.6292
		0	1	2		
Lesson I, Post-instruction task*	High	5	3	8	1.1875	.9106
	Low	7	5	4	.8125	.8342
		0	1	2		
Lesson II, Problem 1*	High	4	1	11	1.4375	.8921
	Low	7	1	8	1.0625	.9979

Table entries represent number of subjects

*Tasks on which between group differences were predicted.

		0	1	2		
Lesson II, Problem 2*	High	1	2	13	1.7500	.5774
	Low	5	1	10	1.3125	.9465

		0	1	2		
Lesson II, Post-instruction task*	High	3	2	11	1.5000	.8165
	Low	6	2	8	1.1250	.9574

		0	1	2		
Lesson III, Problem 1	High	0	0	16	2.0000	.0000
	Low	0	0	16	2.0000	.0000

		0	1	2		
Lesson III, Problem 2	High	1	8	7	1.3750	.6191
	Low	1	8	7	1.3750	.6191

		0	1	2		
Lesson III, Problem 3	High	1	9	6	1.3125	.6021
	Low	1	13	2	1.0625	.4425

		0	1	2		
Lesson III, Post-instruction task	High	5	8	3	.8750	.7188
	Low	9	3	4	.6875	.8732

		0	1	2		
Lesson IV, Problem 1*	High	1	0	15	1.8750	.5000
	Low	4	0	12	1.5000	.8944

		0	1	2		
Lesson IV, Problem 2*	High	0	1	15	1.9375	.2500
	Low	1	4	11	1.6250	.6191

		0	1	2		
Lesson, IV, Post-instruction task*	High	3	3	10	1.4375	.8139
	Low	14	0	2	.2500	.6831

Information Processing Capacity

		0	1	2		
Lesson I, Problem 1	High	0	5	11	1.6875	.4787
	Low	0	5	11	1.6875	.4787

		0	1	2		
Lesson I, Problem 2	High	1	4	11	1.6250	.6191
	Low	2	4	10	1.5000	.7303

		0	1	2		
Lesson I, Post-instruction task*	High	6	3	7	1.0625	.9287
	Low	6	5	5	.9375	.8539

		0	1	2		
Lesson II, Problem 1	High	4	2	10	1.3750	.8851
	Low	7	0	9	1.1250	1.0247

		0	1	2		
Lesson II, Problem 2	High	4	1	11	1.4375	.8921
	Low	2	2	12	1.6250	.7188

		0	1	2		
Lesson II, Post-instruction task*	High	3	3	10	1.4375	.8139
	Low	6	1	9	1.1875	.9811

		0	1	2		
Lesson III, Problem 1	High	0	0	16	2.0000	.0000
	Low	0	0	16	2.0000	.0000

		0	1	2		
Lesson III, Problem 2	High	2	10	4	1.1250	.6191
	Low	0	6	10	1.6250	.5000

		0	1	2		
Lesson III, Problem 3	High	1	14	1	1.0000	.3651
	Low	1	8	7	1.3750	.6191

		0	1	2		
Lesson III, Post-instruction task*	High	7	9	0	.5625	.5123
	Low	7	2	7	1.0000	.9661

		0	1	2		
Lesson IV, Problem 1	High	2	0	14	1.7500	.6831
	Low	3	0	13	1.6250	.8062
		0	1	2		
Lesson IV, Problem 2	High	1	3	12	1.6875	.6021
	Low	0	2	14	1.8750	.3416
		0	1	2		
Lesson IV, Post-instruction task*	High	8	2	6	.8750	.9574
	Low	9	1	6	.8125	.9811

APPENDIX G

MEASUREMENT STRATEGIES USED ON POST-INSTRUCTION TASKS

Measurement Strategies Used on Post-Instruction Tasks

Lesson I

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	2,2,2,7, 7,2,7,7,	6,4 & 6, 7,2,7,7, 5 & 6
	Low	7,4b,1,2, 6,6,7,7	7,2,3,2, 6,2,2,6

Lesson II

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	5,5,8,9, 7,5,8,9	1,1,9,1 7,9,9,9
	Low	9,5,1,9, 1,9,1,9	9,7,9,1, 1,8,8,1

Lesson III

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	5b,5,5c 5a,5a,3 5b,5b	2,1,5b,6 6,5c,6,1
	Low	5&1,1,5b 1,5&1,4 1,5b	5b,1,6,1 6,6,3,6

Lesson IV

		Information Processing Capacity	
		High	Low
Logical Reasoning Ability	High	6c,6c,6b 3,6b,6c 5,6c	6c,6c,6b 6c,6c,6c 6c,5
	Low	6c,3,5,3 3,6c,2,3	5,5,5,5 5,3,5,5

Entries in tables refer to strategy numbers given in the coding schemes, Appendix E.

APPENDIX H

PRETEST PERFORMANCE

Pretest Performance

Eight items were included in the pretest. The items and the criteria used for scoring performance are described in Appendix B. Point counting was scored as successful (S) or unsuccessful (U). Cumulative performance on length conservation and length transitivity was scored as successful (S), transitional (T), or unsuccessful (U). Performance on each of the measurement tasks was scored as successful (S), partially successful (P), or unsuccessful (U). High (H) backward digit span was considered to be a span of three or more and low (L) backward digit span was considered to be a span of two or less.

In order to reduce testing time and to make minimal demands on students and teachers, pretesting with a particular subject was concluded when the subject was eliminated from the final sample (see pp. 117-120 for a description of the criteria used for sample selection). Consequently, many potential subjects received only some of the pretest items. A (-) indicates that the subject did not receive that item.

The following is a summary of pretest performance using the abbreviations described above.

Subject Number	Point Counting	Length conservation/ Length transitivity	Measurement				Backward Digit Span
			1	2	3	4	
School A							
1	S	U	P	-	-	-	-
*2	S	U	U	U	U	U	L
3	S	T	-	-	-	-	-
4	S	U	P	-	-	-	-
5	S	T	-	-	-	-	-
6	S	S	S	-	-	-	-
*7	S	U	U	U	U	U	H
8	S	T	-	-	-	-	-
9	S	T	-	-	-	-	-
10	S	U	S	-	-	-	-
*11	S	U	U	U	U	U	L
12	S	T	-	-	-	-	-
13	S	S	U	U	U	S	-
*14	S	U	U	U	U	U	L
15	S	T	-	-	-	-	-
16	S	T	-	-	-	-	-
*17	S	U	U	U	U	U	H
18	S	U	U	P	-	-	-
*19	S	U	U	U	U	U	L
*20	S	U	U	U	U	U	H

Subject	Point	Length conservation/	Measurement				Backward
Number	Counting	Length transitivity	1	2	3	4	Digit Span
*21	S	U	U	U	U	U	H
22	S	S	U	P	-	-	-
23	S	U	U	U	P	-	-
*24	S	U	U	U	U	U	L
*25	S	S	U	U	U	U	H
26	S	T	-	-	-	-	-
*27	S	U	U	U	U	U	L
28	S	U	U	P	-	-	-
29	S	T	-	-	-	-	-
30	S	S	S	-	-	-	-
31	S	T	-	-	-	-	-
*32	S	U	U	U	U	U	L
33	S	U	U	U	S	-	-
34	S	U	S	-	-	-	-
35	S	T	-	-	-	-	-
School B							
36	S	S	U	S	-	-	-
37	S	U	U	P	-	-	-
38	S	S	S	-	-	-	-
39	S	T	-	-	-	-	-
*40	S	U	U	U	U	U	H

Subject	Point	Length conservation/	Measurement				Backward
Number	Counting	Length transitivity	1	2	3	4	Digit Span
*41	S	S	U	U	U	U	L
42	S	T	-	-	-	-	-
43	S	U	S	-	-	-	-
*44	S	U	U	U	U	U	H
45	S	U	U	P	-	-	-
*46	S	S	U	U	U	U	H
47	S	T	-	-	-	-	-
*48	S	U	U	U	U	U	H
49	S	T	-	-	-	-	-
*50	S	U	U	U	U	U	L
51	S	T	-	-	-	-	-
52	S	U	U	U	U	U	L
53	S	T	-	-	-	-	-
54	S	S	S	-	-	-	-
55	S	U	P	-	-	-	-
56	S	T	-	-	-	-	-
57	S	T	-	-	-	-	-
*58	S	S	U	U	U	U	L
59	S	S	S	-	-	-	-
*60	S	U	U	U	U	U	H
61	S	U	-	-	-	-	-

Subject Number	Point Counting	Length conservation/ Length transitivity	Measurement				Backward Digit Span
			1	2	3	4	
62	S	T	-	-	-	-	-
63	S	U	-	-	-	-	-
64	S	U	-	-	-	-	-
65	S	S	U	U	U	S	-
66	S	U	-	-	-	-	-
*67	S	S	U	U	U	U	L
68	S	U	-	-	-	-	-
69	S	T	-	-	-	-	-
*70	S	S	U	U	U	U	L
71	S	T	-	-	-	-	-
*72	S	S	U	U	U	U	H
73	S	T	-	-	-	-	-
74	S	T	-	-	-	-	-
75	S	U	-	-	-	-	-
76	S	U	-	-	-	-	-
*77	S	S	U	U	U	U	L
78	S	U	-	-	-	-	-
79	S	U	-	-	-	-	-
80	S	T	-	-	-	-	-
81	S	U	-	-	-	-	-
82	S	U	-	-	-	-	-

Subject	Point	Length conservation/	Measurement				Backward
Number	Counting	Length transitivity	1	2	3	4	Digit Span
83	S	T	-	-	-	-	-
84	S	T	-	-	-	-	-
85	S	T	-	-	-	-	-
86	S	U	-	-	-	-	-
87	S	U	-	-	-	-	-
88	S	U	-	-	-	-	-
89	S	U	-	-	-	-	-
90	S	T	-	-	-	-	-
91	S	S	U	U	U	S	-
92	S	U	-	-	-	-	-
*93	S	S	U	U	U	U	L
94	S	U	-	-	-	-	-
95	S	T	-	-	-	-	-
96	S	T	-	-	-	-	-
97	S	T	-	-	-	-	-
98	S	U	-	-	-	-	-
*99	S	S	U	U	U	U	H
100	S	T	-	-	-	-	-
101	S	T	-	-	-	-	-
102	S	S	S	-	-	-	-
103	S	U	-	-	-	-	-

Subject Number	Point Counting	Length conservation/ Length transitivity	Measurement 1	2	3	4	Backward Digit Span
104	S	U	-	-	-	-	-
105	S	T	-	-	-	-	-
School C							
106	S	U	-	-	-	-	-
*107	S	S	U	U	U	U	L
108	S	U	-	-	-	-	-
109	S	S	U	S	-	-	-
110	S	T	-	-	-	-	-
111	S	U	-	-	-	-	-
112	S	T	-	-	-	-	-
113	S	T	-	-	-	-	-
*114	S	S	U	U	U	U	L
115	U	-	-	-	-	-	-
116	S	U	-	-	-	-	-
117	S	U	-	-	-	-	-
118	S	T	-	-	-	-	-
*119	S	S	U	U	U	U	H
120	S	U	-	-	-	-	-
121	S	S	U	S	-	-	-
122	S	U	-	-	-	-	-
123	S	T	-	-	-	-	-

Subject Number	Point Counting	Length conservation/ Length transitivity	Measurement 1	2	3	4	Backward Digit Span
*124	S	S	U	U	U	U	H
*125	S	S	U	U	U	U	H
126	S	U	-	-	-	-	-
127	S	T	-	-	-	-	-
128	S	U	-	-	-	-	-
129	S	T	-	-	-	-	-
130	S	U	-	-	-	-	-
131	S	U	-	-	-	-	-
132	S	S	S	-	-	-	-
133	S	T	-	-	-	-	-
134	S	T	-	-	-	-	-
135	S	T	-	-	-	-	-
136	S	S	U	U	U	S	-
*137	S	S	U	U	U	U	H
138	S	U	-	-	-	-	-
139	S	U	-	-	-	-	-
140	S	U	-	-	-	-	-
141	S	U	-	-	-	-	-
142	S	U	-	-	-	-	-
143	S	T	-	-	-	-	-

*Subjects selected for final sample

Center Planning and Policy Committee

Richard A. Rossmiller
Wayne Otto
Center Co-Directors

Dale D. Johnson
Area Chairperson
Studies in Language:
Reading and Communication

Marvin J. Fruth
Area Chairperson
Studies in Implementation
of Individualized Schooling

Penelope L. Peterson
Area Chairperson
Studies of Instructional Programming
for the Individual Student

James M. Lipham
Area Chairperson
Studies of Administration and
Organization for Instruction

Thomas A. Romberg
Area Chairperson
Studies in Mathematics and Evaluation
of Practices in Individualized Schooling

Associated Faculty

Vernon L. Allen
Professor
Psychology

B. Dean Bowles
Professor
Educational Administration

Thomas P. Carpenter
Associate Professor
Curriculum and Instruction

W. Patrick Dickson
Assistant Professor
Child and Family Studies

Lloyd E. Frohreich
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